FOUNDATION 'DIVE RESEARCH'

AIR PRESSURE AND TEMPERATURE IN A DIVE BOTTLE DURING DIVING

H. J. van Grol
Summary

This report is composed in response to some discussions within the board of the foundation 'Dive Research', on the matter what the temperature of a dive bottle and the temperature and pressure of the contained air therein, will be during diving to for instance a depth of 20 m. A first global assessment readily produces answers of little value, due to the large amount of independent variables/parameters as well as of uncertainties.

Answering the above quoted question, proves to be only useful if the problem at hand can be based on a set of retrievable models, describing the concerned physiological and physical phenomena. Further, in order to get some feeling of the implication of the uncertainties with which the determining quantities are established, the numerical elaboration must be focussed on a parametric approach. Aspects, which influence the outcome to a large degree, are:

i. the breathing and swimming history, before actually going for a dive, such as:
   - does functional testing of the breathing apparatus by the diver takes place before going to swim and/or to descend?;
   - as above but now with regard to swimming at what speed and how long, without and/or with breathing from the dive set;

ii. the required amount of breathing flow (inspiration volume) for a specific amount of work, viz. swimming velocity at surface, during descend or at a certain (bottom) depth;

iii. the numerical quantification of the heat transfer capacity between sea-water and dive bottle through forced convection, which is definitely not very well defined, not to say hampered by the carrier-set of the dive bottle and the wake behind the diver himself;

iv. the largely unpredictable process of natural heat transfer between dive bottle and contained air, once more complicated by the variable inclination angle of the dive bottle; surface swimming, descending, etc.

The analytical model based concept is of an elementary engineering composition, though the numerical completion requires a terrific perseverance, as the concerning common differential equations are of a non-linear nature, brought about by non-constant coefficients. Three different dive scenarios are analysed. The first merely to show that the model approach does make sense and that results and trends can be conceived.

For the reader not familiar in handling/using analytical expressions, the author tried to relieve this mathematical shortcoming by introducing, whenever possible numerical examples.

The conclusive results and conclusions are summarized below.

1. The breathing model is set up as a series of successive instantaneous inhalations in function of the swimming velocity and dive profile.

2. The instantaneous acts of breathing/inhalation volumes will allow the corresponding air pressure and temperature decreases in the dive bottle, to compute as an adiabatic process.

3. An engineering elliptical based interpolation model is constructed by the author to establish the natural heat convection transfer between dive bottle and contained air in function of among others the inclination angle of the dive bottle.

4. The model based overall approach proves to be useful to predict/quantify the temperature changes of a dive bottle as well as the temperature and pressure changes of the contained air therein, during different dive scenarios, including those with a realistic pre-phase before descending.

5. The influence of the heat transfer capacity/process between dive bottle and contained air on the temperature progress of the dive bottle during pre-phase and/or diving is about nil, implying among others that the uncertainties in the heat transfer between bottle and dive air on the accuracy of the temperature development of the dive bottle can be neglected all together.
6. The half-value time of the temperature progress of the dive bottle during descend with a swimming velocity of the diver of 23 m/min is to the author's best knowledge $35^{\pm 15}$ s. This turns in $65^{\pm 25}$ s, in case of the realistic dive scenario III, with a pre-dive phase of 90 s.

7. With regard to dive air, one could not speak of a specific half-value time, but only of momentary half-value times ranging from a minimum value to infinity. Characteristic minimum value for the scenarios II and III are in the order of two to three minutes.

8. It is estimated that the semi asymptotic stationary dive air temperature is eight °C below the ambient sea-water temperature of 26 °C, assuming that the diver swims at a depth of 20 m with a velocity of 12 m/min and that the initial pressure at the onset of going for diving is 200 bar.

This temperature difference is dependent on the air pressure in the dive bottle, approximately inversely proportional to $\sqrt{P_a(t)}$. For the case of an initial air pressure of 100 bar the corresponding temperature difference increases to 13 to 14 °C.

9. Heat transfer to and from the contained dive air is modelled, under the assumption that heat conduction through the shell wall of the dive bottle is infinite. This simplification does not effect any of the results as other uncertainties certainly outstrip this shortcoming.

10. The breathing model is in need of an adjustment for the influence of the increasing specific air mass density, $\rho_a$ with diving depth. An attempt of the author to come across a trail how to implement this by justifying why the MBC (Maximum Breathing Capacity) is inversely proportional to $\sqrt{\rho_a}$ approximately, failed. Nevertheless, the present results will not be influenced very much as all scenarios are limited to a depth of 20 m only. For sure at greater depth the breathing model must be accommodated for this effect, possibly by the introduction of a proper pulmonary hysteresis effect.
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1 INTRODUCTION

This report\(^1\) is composed in order to obtain insight in the change in temperature of a dive bottle and the change in temperature and pressure of the air in that bottle, during skin diving. Such a question was raised along discussions among members of the board of the foundation 'Dive Research', with regard to what accuracy the amount of consumed dive air can be computed from the air pressure decrease in a dive bottle, during actual diving. Specific interest concerns a dive to a depth of 20 m with a descend speed of 20m/min.

The large number of parameters as well as the large uncertainties in heat transfer coefficients has been the reason to solve this problem on basis of a set of well defined models.

Uncertainties and parameters are:
- the relative large variation in respiratory minute volume (RMV) in dependence of both the work/swimming output and dive depth/profile;
- the moderately up to badly quantifiable state of flow around a dive bottle during surface swimming, the descend to 'bottom depth' and subsequently horizontal swimming at depth, in particular brought about by:
  - the influence of the bottle carrier set on the flow pattern around the dive bottle;
  - as above but now with regard to the disturbing wake effects behind the diver;
- the heat transfer process between dive bottle and its ambiance (air or water) by forced convection and that between dive bottle and the contained air through natural convection, thereby accounting for the position/inclination of the air bottle.

The model based analytical approach concerns the:
- description of the model principle; chapter 2;
- determination of the relation between breathing flow (RMV) on the one hand and the swimming velocity and diving depth on the other hand; chapter 2 and 3;
- formulation, analysis and numerical solution methodology of the heat transfer process between dive bottle and its ambience and between dive bottle and the contained air therein; chapter 3;
- determination of the air temperature and pressure drop within the dive bottle on basis of an adiabatic flow process as a result of a prescribed breathing volume; chapter 3.

Chapter 4 covers the justification of the numerical data, in particular the quantification of the heat transfer coefficients.

Chapter 5 contains the results, commented by means of a triad dive scenarios. The third scenario includes surface swimming and - after descend - swimming at a certain depth.

Finally, the conclusive results and conclusions are compiled in chapter 6.

\(^{1}\) The Dutch draft version of this report 'SDR_Temp bottle en lucht slotvs [stick-02]' is issued in September 2003.
2 DESCRIPTION OF MODELLING

2.1 Model principle

General

Assume that at the moment in time a scuba dive commences, the dive bottle with its contents has reached a certain equilibrium temperature denoted as $T_{\text{amb}} \, [{\degree}\text{C}]$. The ambient temperature will change when the diver takes the dive. Generally, the sea-water temperature, $T_{\text{w}} \, [{\degree}\text{C}]$ will be lower than the initial ambient air temperature. Or, with other words at the beginning of a dive, the dive bottle will cool down as a result of heat transfer from bottle to water.

During breathing however the air temperature within the dive bottle will decrease at each inhalation due to adiabatic air depressurisation. The air pressure and subsequently the temperature drop will soon be that large that heat will be transferred from sea-water via the air bottle to the air within the bottle.

The simultaneously occurrence of breathing and heat transfer is simplified in such a way that inhaling occurs within an infinite short length of time per each respiratory cycle.

Model description

It is assumed that the process of inspiration takes place at an infinitesimal short length of time between two breathing cycles. In between two instantaneous inhalations heat transfer occurs between sea-water, dive bottle and the air within the bottle; see for the temperature development in function of time, $T(t)$ fig. 01 below under $\Delta$. It is further assumed that the air temperature/pressure drop in the dive bottle due to the instantaneous act of inhaling is a pure adiabatic process; see $\triangledown$. The course of time is therefore discretized in a number of breathings $i$.

\[
t_i \, \text{[min]} = i \cdot T_{\text{resp}, i} \, \text{[s]} / 60 \quad (2-01)
\]

in which: $T_{\text{resp}, i}$ [s] period of one breathing cycle $i$ [-] number of breathings (breathing index) ; $i$: 0, 1, 2, ...

The progress of the above described process is depicted schematically in fig.01 below.

---

Fig. 01  Separation between the course of the heat transfer process and the instantaneous act of inhalation.

1 Between the points in time $i(\ +)$ and $i + 1 \ (-)$ heat transfer takes place and no breathing

- $T_{\text{bott}}$: $T_{b,i}$ $\rightarrow$ $T_{b,i+1}$
- $T_{\text{air}}$: $T_{\text{a},i(\ +)}$ $\rightarrow$ $T_{\text{a},i+1(\ -)}$
- $P_{\text{air}}$: $P_{a,i(\ +)}$ $\rightarrow$ $P_{a,i+1(\ -)}$
- $C_{1,i+1}$ and $C_{2,i+1}$ [$^\circ$C] on $i$ dependent "integration constants"

2 adiabatic air depressurization takes place at the moment in time $t_i$

- $T_{\text{bott}}$: $T_{b,i(\ +)}$ $\rightarrow$ $T_{b,i(\ +)}$
- $T_{\text{air}}$: $T_{\text{a},i(\ +)}$ $\rightarrow$ $T_{\text{a},i(\ +)}$
- $P_{\text{air}}$: $P_{a,i(\ +)}$ $\rightarrow$ $P_{a,i(\ +)}$

$T$ [$^\circ$C] temperature, in case of $T_b$ and $T_a$, otherwise in case of $T_A$: [K]

$P$ [bar] pressure

$i(\ +)$ [-] refers to the moment in time: $t_i(\ +) = \lim_{\Delta t \to 0} \rightarrow 0 , \text{for } \Delta t > 0 (t = t_i + \Delta t)$

$i(\ -)$ [-] refers to the moment in time: $t_i(\ -) = \lim_{\Delta t \to 0} \rightarrow 0 , \text{for } \Delta t > 0 (t = t_i - \Delta t)$
2.2 Breathing model

On the rate of breathing volume and swimming velocity

\[ V_{\text{tot}} \] total lung capacity
\[ V_{\text{work}} \] tidal volume, if constant during descending, otherwise \( V_{\text{work}, i} \)
\[ V_{\text{res}} \] residual volume
\[ V_{\text{urv}} \] unused reserve volume; \( V_{\text{urv}} = V_{\text{tot}} - V_{\text{work}} \geq V_{\text{res}} \) (2-02)

The performance of a certain continuous amount of work - in the sense of external output - is primarily related to the corresponding required oxygen consumption, denoted as Oxygen Minute Value, OMV [nℓ/min; at 1 bar and 0 °C]. Based on the measuring data given in [01] fig. 1-26, the following relation between OMV and the swimming speed of a skin diver is established:

\[
\text{OMV} [\text{nℓ/min}] = 0.50 + 1.01 \cdot 10^{-3} \cdot V_{d}^{2.1} \quad 0 \leq V_{d} \leq 35 \text{ m/min} \quad (2-03)
\]

\[
\text{OMV}_{\text{spread}} \pm \{0.18 + 0.025(V_{d} - 16)\} \quad 16 \leq V_{d} \leq 35 \text{ m/min} \quad (2-03\text{-a})
\]

\[ V_{d} \quad [\text{m/min}] \text{ swimming velocity (underwater)} \]

Justification and explanation

i. The above-mentioned eqs. (2-03) are based on nine, as averaged determined data points of "good swimmers" and an other group of underwater swimmers, all of them swimming within the interval of \( 16 \leq V_{d} \leq 35 \text{ m/min} \). The swimming depth is not mentioned; presumed is a depth of 3 m (~ 10 ft). In addition, the spread is not further declared than as: "range, both groups".

ii. Eq. (2-03) is made applicable for an extended interval of \( 0 \leq V_{d} \leq 35 \text{ m/min} \), while the data points of [01], on which the equation is based refer to the interval of \( 16 \leq V_{d} \leq 35 \text{ m/min} \).

The value of OMV at \( V_{d} = 0 \text{ m/min} \) is chosen as the value of the arterial partial pressure for carbon dioxide is kept constant at 40 mmHg, independent of diving depth. Thus:

\[ \text{RMV} \text{ [U/min]} \]

With other words these deviations are negligible compared to the spread in measuring results shown in [01], expressed here as eq. (2-03-a).

iv. That the OMV's show a \( V_{d}^{2\text{-dependence}} \), is caused by the swimming drag, which is proportional to the squared swimming velocity in case of a turbulent flow at Reynolds-numbers of:

\(~ 5 \cdot 10^3 \text{ to } 10^6\), see for instance [02]. At \( V_{d} = 20 \text{ m/min} \), a sea-water temperature of 20 °C, a characteristic length of 0.6 m, \( \rho \text{seawater}, 3\% \text{ sal} = 1.024 \text{ kg/dm}^3 \) en \( \eta \text{seawater}, 20 \text{ °C} = 1.01 \cdot 10^{-3} \text{ Pa.s} \), it is found that:

\[ \text{Re} = 2. \cdot 10^5 \text{.} \]

At low swimming velocities, for \( \text{Re} < 10^4 \) the turbulent flow turns into a laminar flow. In those cases, a \( V_{d}^{2} \) instead of a \( V_{d}^{2\text{-dependence}} \) prevails. The concerned velocities are however so low (< 1 m/min), that the drag effect - laminar instead of turbulent flow - is negligible in comparison with the constant; 0.50 term in eq. (2-03).

v. The fact that the power to which \( V_{d} \) in eq. (2-03) is raised is a bit larger than 2, is explicable. After all, in order to generate a certain external power output, a corresponding muscle work of the breathing system has to discount for in the OMV as well.

The potential oxygen supply capacity is determined by the combined working of the blood circulation system - hydraulic pump system, heart, vessels, et cetera - and the breathing system - pneumatic system, lungs, chest muscle system, et cetera - on the one hand and the interfacing system being the oxygen-carbon dioxide exchange capacity within the alveoli sacs on the other hand. Proper control of the oxygen-carbon dioxide balance in the pulmonary veins is obtained by keeping the partial pressure of carbon dioxide in the arteries, viz. in the alveoli constant. On basis of this modelling, it is estimated that the required Respiratory Minute Value, RMV [U/min] is in the order of 22 times that of OMV.

Justification

From the point of view of breath-control, we breathe primarily in order to get rid of the excess of carbon dioxide. This is arranged in such a way that the set-value for the arterial partial pressure for carbon dioxide is kept constant at 40 mmHg, independent of diving depth. Thus:

\[ V_{d} \quad [\text{m/min}] \text{ swimming velocity (underwater)} \]

---

The OMV-values at 0.5 en 1.2 knot in [01] published in 1963 are the same as those in [01] fig. 3-6 published in 2005.
\( P_{\text{part. CO}_2} = P_{\text{alv. CO}_2} \)
\( P_{\text{part. CO}_2} = 40 \text{ mm Hg} \) (physiological set value) \( \)  
\( P_{\text{amb.}} = (1 + D[\text{m}] / 10) \cdot 760 \text{ mm Hg} \) \( \) (c)

The ventilation of the alveoli sacs is modelled as depicted below.

In case of a stationary state of gas exchange:

\[
\frac{P_{\text{alv. CO}_2}}{P_{\text{amb.}} - P_{\text{alv. H}_2O}} = \frac{CMV[n \ell/min] + (1 + D[\text{m}] / 10) \cdot RMV_{\text{alv.}}[p\ell/min] \cdot P_{\text{CO}_2, \text{air}} [-]}{(1 + D[\text{m}] / 10) \cdot RMV_{\text{alv.}}[p\ell/min]} \tag{d}
\]

RMV and RMV\text{alv} are related as follows:

\[
RMV[p\ell/min] = RMV_{\text{alv.}}[p\ell/min] + V \_\text{d. s.} \cdot v_b \tag{e}
\]

in which:

- V\_d.s. \text{ [\ell]} dead space
- v_b \text{ [min\(^{-1}\)}] breathing frequency
- p_{\text{CO}_2, \text{air}} \text{ [%]} CO\_2 content of supplied dive air

Combining the eqn. (c) up to (e) inclusive and making use of \( RQ \equiv \frac{CMV}{OMV} \), results in:

\[
RMV[p\ell/min] = V \_d.s. v_b + \frac{P_{\text{alv. CO}_2}[\text{Hg}]}{760 - P_{\text{alv. H}_2O}[\text{Hg}] / (1 + D[\text{m}] / 10)} - 0.01 \cdot (1 + D[\text{m}] / 10) \cdot p_{\text{CO}_2, \text{air}} [-%] \tag{d}
\]

Numerical data:

- \( P_{\text{alv. H}_2O} = 47 \text{ mm Hg} \)
- \( V \_d.s. \approx 0.25 \ell \)
- \( V_{\text{alv. CO}_2} = 40 \text{ mm Hg} \)
- \( v_b \approx 17 \text{ min}^{-1} \)
- \( RQ = 0.90 \)
- \( p_{\text{CO}_2, \text{air}} \approx 0.04 \% \)

Eq. (d) turns in:

\[
RMV[p\ell/min] \approx 4.25 + \frac{17.1 \cdot OMV[n\ell/min]}{1 - 0.0076(1 + D[\text{m}] / 10) + 0.062 \cdot (1 + D[\text{m}] / 10)^{-1}} \tag{e}
\]

### Numerical result

<table>
<thead>
<tr>
<th>Depth [m]</th>
<th>RMV[p\ell/min] / OMV[n\ell/min]</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>20.3</td>
</tr>
<tr>
<td>10</td>
<td>21.1</td>
</tr>
<tr>
<td>20</td>
<td>21.4</td>
</tr>
<tr>
<td>40</td>
<td>21.8</td>
</tr>
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In other words it is conceivable that \( RMV[p\ell/min] / OMV_{\text{alv.}}[n\ell/min] \approx 21 [-] \)

With the data compiled in [01] table 1-3, supplemented/confirmed by measured data from [06] fig. 5-9 (a copy of this figure is shown as fig. 03), the following empirical correlation between OMV and RMV is obtained for OMV-values up to 3 to 4 n\ell/min:

\[
RMV \sim (22\frac{1}{2} \pm 1.5) \text{ OMV} \quad \text{OMV} < 3 \text{ to } 4 \text{n\ell/min} \rightarrow \text{RMV} < 70 \text{ to } 90 \text{n\ell/min} \tag{2-04}
\]

Combining the eqn. (2-03) and (2-04) and taken care of the proper use of [p\ell/min] versus [n\ell/min] and of the time indices, the following result is obtained:

\[
\begin{align*}
RMV, [p\ell/min] & = 11.3 + 0.0227 \cdot (v_{d,\text{t-1}})^{2.1} \quad 0 \leq v_{d,\text{t-1}} \leq 35 \text{ m/min} \tag{2-05} \\
RMV, \text{spread,}_{\text{t}} & \pm \{4.1 + 0.56 (v_{d,\text{t-1}} - 16)\} \quad 16 \leq v_{d,\text{t-1}} \leq 35 \text{ m/min}
\end{align*}
\]

\(^3\) Vapor pressure at a body temperature of \( T_{\text{alveoli}} = 37 \degree \text{C} \)
A graphical representation of these equations is given as fig. 02.

**Some informative numerical data, among others**

<table>
<thead>
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<th>obtained from [01] table I-3</th>
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<tbody>
<tr>
<td>RMV, bed rest ~ 6 ℓ/min;</td>
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<tr>
<td>RMV, slow walking; 3.21 km/hr</td>
</tr>
<tr>
<td>RMV, fast walking; 6.44 km/hr</td>
</tr>
<tr>
<td>RMV, running; 12.9 km/hr</td>
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<tr>
<td>RMV, diver v = 30.9 m/min = 1 knot</td>
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obtained by eq. (2-05) and
according to [07]: dV\(E\)/dt ~ 41 ℓ/min.

**Redundant dependent variables**

The variables RMV, \(V_{work}\) and \(T_{resp}\) are mutually correlated as follows:

\[ V_{work} \times v_b = RMV \]  \hspace{1cm} (2-06)

\[ v_b \times T_{resp} = 60 \] \hspace{1cm} (2-06-a)

in which: \(v_b\) [min\(^{-1}\)]: breathing frequency

Or in other words for a certain swimming velocity the required work output in terms of RMV is given by (eq. (2-05)). This implies that either \(v_b\) (or \(T_{resp}\)) or \(V_{work}\) can be chosen as a parameter. See table 03 for a numerical example.

Some physiological lung performance baseline data is compiled in table 01; it concerns adapted data obtained from [06] table 5-1 based on more than 25 test persons. The content of this document, however does not deal with short durance top performances of say some ten seconds but with common sport diving practises. As the relation between RMV and the required power output (thus the corresponding OMV) will not be linear up to their maximum values, the question arises to which value of \(V_{tidal}\) (≡ \(V_{work}\)) this might be true?

The author adopted for the ratio of \(V_{tidal}\) linear relation between RMV and OMV over \(V_{tidal, max}\) a value of \(\frac{2}{3}\). Above this value, the RMV’s will increase exponentially with the required work output (load).

**Numerical example**

Assume a skin diver swims horizontal with a swimming velocity of \(v_d = 25\) m/min.

With eq. (2-05) it is found that: RMV = 31 ℓ/min.

Referring to table 01 (processed data from [06] table 5-1), \(V_{work}\) is found to be ~ 2.2 for men ~ 1.5 for women. By making use of the eqs. (2-06) and (2-06-a) the following result is obtained:

- men: \(v_b = 14\) /min and \(T_{resp} \sim 4\) s
- women: \(v_b = 21\) /min and \(T_{resp} \sim 3\) s

These values are in every way credible, indeed.

**Clarification**

In fig 03 the pulmonary ventilation flow, RMV is plotted against the oxygen demand (uptake), OMV. It is clearly seen that the efficiency, that is to say the capacity of the breathing mechanism decreases strongly at values of OMV beyond 3 to 4 ℓ/min. The increased need of oxygen goes along with an

\[ \frac{dV_{O,2}}{dt} = 1.7\ \text{ℓ/min}, \quad P_{A,C0} = 35\ \text{mm Hg}\ \text{en} \quad V_D = 0.1\ V_A \]

\[ \text{Page 74, with} \quad dV_{O,2}/dt = 1.7\ \text{ℓ/min}, \quad P_{A,C0} = 35\ \text{mm Hg}\ \text{en} \quad V_D = 0.1\ V_A \]

\[ \text{Page 74, with} \quad dV_{O,2}/dt = 1.7\ \text{ℓ/min}, \quad P_{A,C0} = 35\ \text{mm Hg}\ \text{en} \quad V_D = 0.1\ V_A \]

\[ \text{Page 74, with} \quad dV_{O,2}/dt = 1.7\ \text{ℓ/min}, \quad P_{A,C0} = 35\ \text{mm Hg}\ \text{en} \quad V_D = 0.1\ V_A \]

\[ \text{Page 74, with} \quad dV_{O,2}/dt = 1.7\ \text{ℓ/min}, \quad P_{A,C0} = 35\ \text{mm Hg}\ \text{en} \quad V_D = 0.1\ V_A \]

\[ \text{Page 74, with} \quad dV_{O,2}/dt = 1.7\ \text{ℓ/min}, \quad P_{A,C0} = 35\ \text{mm Hg}\ \text{en} \quad V_D = 0.1\ V_A \]
'explosive' increase in the rate of air request (breathing volume per minute), finally ending up with an asymptotical value of the oxygen uptake at the maximum value of air ventilation (MBC: Maximum Breathing Capacity).

**Note**
If the validity range of OMV for which eq. (2-04) is valid is inadequately than eq. (2-04) can be restructured as:

\[ RMV \sim (22^{1/2} \pm 1.5) \cdot OMV + \beta / (OMV_{\text{max}} - OMV) \]  

in which:
- \( OMV \leq OMV_{\text{max}} \)
- \( OMV_{\text{max}} \approx 5.0^{0.5} \text{ ℓ/min} \)
- \( \beta \approx 3.5 \text{ ℓ/min}^2 \)

For top sportsmen the maximum value of OMV can amount to:
- \( OMV_{\text{max}, 10 \text{ tot } 15 \text{ min}} = 5 \text{ to } 6 \text{ ℓ/min} \)
- \( RMV_{\text{max}, 10 \text{ tot } 15 \text{ min}} = 110 \text{ to } 130 \text{ ℓ/min} \)

**Quote from ref. [06] that goes with figure 5-9 (present fig. 03):**
"Pulmonary ventilation at rest and during exercise (running and cycling). Four individual curves are presented. Several rates of exercise gave the same maximum oxygen uptake. Exercise time from 2 to 6 min. Stars denote individual values for top athletes measured when maximum oxygen uptake was attained. …"
In ref. [07], page 34 et cetera it is stated that this dependency is attributed to the fact that a flow velocity in an air channel - thus the corresponding RMV in the airways - shall never exceed the propagation speed of channel wall deformation.

**Further model considerations**

**i.** It is remarkable that there is no obvious simple explanation for the approximate dependency of MBC on $1/\rho$. In order to obtain some insight in the possible cause, a number of flow models will be scrutinized here beneath.

**ii.** Assume that the breathing system - trachea up to alveoli sacs inclusive⁶ - consists of one schematized flow tube only, with length $L$ and diameter $D$.

The power, $P_l$, to maintain a certain volume flow in the said flow tube equals:

$$P_l \ [\text{Nm/s}] = d(\text{work})/dt = d(\text{force \times distance})/dt = \tau \cdot \pi \cdot D \cdot dL \cdot L/dt = \tau \cdot \pi \cdot D \cdot L \cdot v_{av.}.$$ (a)

in which: $\tau \ [\text{N/m}^2]$ : shear stress, which the tube wall exerts on the flow.

Force equilibrium yields:

$$\Delta p_l = \tau \cdot \pi \cdot D \cdot L / (1/4 \pi \cdot D^2)$$ (b)

in which: $\Delta p_l \ [\text{N/m}^2]$ : pressure drop along a tube wall distance $L$.

Combination of the eqs. (a) and (b), results in:

$$P_l = \Delta p_l \cdot \psi_v$$ (c)

$$\psi_v = 1/2 \cdot \pi \cdot D^2 \cdot v_{av.}$$ (d)

in which: $\psi_v \ [\text{m}^3 / \text{s}]$ : volume flow, ...

For different flows it is found that, see for instance ref. [02]:

- laminar flow:
  $$\Delta p_l = (64/Re) \cdot (L/D) \cdot 1/2 \cdot \rho \cdot (v_{av.})^2$$ (e)

- turbulent flow, smooth tube:
  $$\Delta p_l = (0.316/Re^{0.5}) \cdot (L/D) \cdot 1/2 \cdot \rho \cdot (v_{av.})^2$$ (f)

- turbulent flow, rough tube:
  $$\Delta p_l \approx 0.06 \cdot (L/D) \cdot 1/2 \cdot \rho \cdot (v_{av.})^2$$ (g)

in which: $Re$ [\text{-}]: Reynolds number (see paragraph 4.3)

Combination of the eqn. (c) up to (g) inclusive produces the following result (leaving out the numerical constants and noting that $\eta$ stands for dynamic viscosity; see paragraph 4.3 also):

$$P_{L\text{-laminar}} \approx (L/D^4) \cdot 1/2 \cdot \rho \cdot (v_{av.})^2$$

$$P_{L\text{-turbulent, smooth}} \approx (L/D^{4.75}) \cdot \eta^{0.25} \cdot 1/2 \cdot \rho^{0.25} \cdot (v_{av.})^{2.75}$$

$$P_{L\text{-turbulent, rough}} \approx (L/D^3) \cdot \rho \cdot (\psi_v)^3$$

Note that $\psi_v$ can be interpreted as a measure for RMV. From the right above given set of equations, keeping in mind that the physiological breathing power capacity, $P_l$, is constant (by definition) and the dynamic viscosity $\eta$ is independent of pressure, it follows that:

- laminar flow:
  $$RMV \approx 1/\rho^0$$ [independent on diving depth] (h)

- turbulent flow, smooth tube:
  $$RMV \approx 1/\rho^{0.27} \cdot [0.27 = 0.75 / 2.75]$$ (i)

- turbulent flow, rough tube:
  $$RMV \approx 1/\rho^{1/3}$$ (j)

It is remarkable that for the above mentioned types of flow RMV is inversely proportional to the specific air mass raised to the power 0 or $1/4$ ($0.25$) or $1/3$ respectively, while the experimental results for MBC show a more probable value of $1/2$ (see fig. 04 lower curve).

Also, other flow effects such as sudden contractions/expansions losses - of the type of a orifice plate - in the trachea or further down in the airways do not furnish more insight. In all such cases it turns out that: $\Delta p = \text{constant} \cdot 1/2 \cdot \rho \cdot (v_{av.})^2$, implying that these flow patterns yield the same results as for turbulent flow, rough tube (see eq. (g)).

It is noteworthy to mention that on basis of a schematized lung model, the pressure drop occurs for 75% (model of Weibel) to 80% (model of Olsen) in the upper part of the airways, viz. throat, trachea, bronchus en bronchiole; see ref. [07].

The relation $MBC \equiv RMV_{\text{max}} \approx 1/\rho^{1/2.5} (0.4)$ fits the measurements at best; see last column in table 02 further down. From the physical point of view this implies that at maximum breathing capacity, the corresponding flow drag is not proportional to $1/2 \cdot \rho \cdot v^2$ or $1/2 \cdot \rho \cdot v^{2.4}$ as in eq. (2-03) but to $1/2 \cdot \rho \cdot v^{2.5}$.

---

⁶ In professional literature, [06] en [07] the flow channel is divided in 25 segments, starting with the trachea with two elements and than further down after 22 bifurcations finishing in the alveoli sacs.
Table 02 MBC ratio in function of diving depth

<table>
<thead>
<tr>
<th>Diving depth</th>
<th>Maximum Breathing Capacity Ratio, f_{MBC ratio} [100%]#</th>
</tr>
</thead>
<tbody>
<tr>
<td>[feet]</td>
<td>[m]</td>
</tr>
<tr>
<td>30</td>
<td>9.14</td>
</tr>
<tr>
<td>60</td>
<td>18.3</td>
</tr>
<tr>
<td>90</td>
<td>27.4</td>
</tr>
<tr>
<td>120</td>
<td>36.5</td>
</tr>
<tr>
<td>150</td>
<td>45.7</td>
</tr>
</tbody>
</table>

# At surface f_{MBC ratio} = 100% & Solid curve

Note
A more elegant and conceivable explanation for the relation of RMV with ρ, so as given by eq. (j) goes as follows.
Denote the average pulmonary power as P_{lung}. [Nm/min]. The in- and expired air mass per minute, equals:
m = RMV . ρ_{air} [kg/min]

The required power - kinetic energy to accelerate a mass from 0 to v_{av}. - to in- and expire this mass can be written as (2 corresponds to in-resp.expire):
P ≡ P_{lung} = 2 . ½ m . v_{av}^2 [Nm/min]

RMV can be definition be written as:
RMV = v_{av} . ΔF [U/min]
in which: ΔF [m^2] reference cross section of 'airways'

Combination of the eqn. (k) up to (m) inclusive yields:
P_{lung} = RMV . ρ_{air} . ΔF^2 [Nm/min]

In other words the same breathing power goes along with: RMV ≈ 1 / ρ^{1/3}.

Preliminary conclusions
1. The strong dependency of MBC with diving dept (see fig. 04) is brought about by flow (energy) losses, which appear to be proportional to ρ . v^{2.5} empirically; see table 02, last column. An adequate qualitative explanation is that, this is the combined effect of airflow losses, which are proportional to ρ . v^2 increased with hysteresis losses, in particular if maximum breathing efforts are required.

2. However, the effect of the increased air density with depth on the RMV's during common sport diving activities is not accounted for in this document. The reason is that the author is not familiar with a general accepted explanation - analytical model description - to account for this effect. Therefore, eq. (2-05) is adhered to throughout this document nevertheless the depth the diver swims.

Note
The small influence of swimming depth, brought about by the constant water vapour pressure in the alveoli sacs so as given by eq. (c) in paragraph 2.2 is furthermore disregarded throughout.

2.4 Dive profile
v_{des} [m/min]: descent speed
P_0 [bar]: atmospheric pressure
P_i [bar]: pressure at t= t_i
t_i [min]: time lapse after descend

P_i = P_0 + v_{des} . t_i /10 (2-08)
in which: t_i ≤ D_{bottom} / v_{des}

Making use of eq. (2-01) turns eq. (2-08) in:
P_i = P_0 + v_{des} . i . T_{resp} . 600^{-1} (2-09-a)

Or in case v_{des} or T_{resp} are time dependent variables and accounting for the proper time indices:

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\[ P_i = P_0 + \frac{1}{600} \sum_{j=0}^{j=i-1} v_{\text{des},j} T_{\text{resp},j} \]  

(2-09-b)

If the diver descends with a diving speed \( v_d \) at an angle \( \alpha \) with the vertical, than:

\[ v_{\text{des}} = v_d \cdot \cos \alpha \]  

(2-10)

\[ P_i \text{ [bar]} = P_0[\text{bar}] + v_d[\text{m/min}] \cdot \cos \alpha \cdot i \cdot T_{\text{resp}}[\text{s}] \cdot 600^{-1} \]  

(2-11)

**Numerical example**

Given:

A perfect trimmed diver dives at an angle of 30° with the vertical to a depth of 20 m with a swimming speed of 20 m/min.

Question:

Required RMV and total air consumption?

Answer:

\[ v_{\text{des}} = 20 \text{ m/min} \rightarrow \text{eq. (2-10): } v_d = 20/\cos30 = 23.1 \text{ m/min} \rightarrow \text{eq. (2-05): } \text{RMV} = 28^{\text{th}} \text{pl/min} \]

Chose \( T_{\text{resp}} \) as a parameter (see earlier paragraph 2.2 under 'Redundant dependent variables'), then the total consumed air volume will be ~58 ℓ/min for all three chosen values of \( T_{\text{resp}} \); see table 03 below. This is obvious as both RMV as well as \( t_{\text{descent}} \) are the same for all three cases.

**Table 03** Air consumption per inspiration during a dive to 20 m with \( T_{\text{resp}} \) as parameter.

<table>
<thead>
<tr>
<th>( T_{\text{resp}} ) [s]</th>
<th>( v_b ) [-/min]</th>
<th>( t_{\text{inspiration}} ) [s]</th>
<th>( V_{\text{work}} ) = RMV/( v_b ) [pℓ/inspiration]</th>
<th>( P_i ); eq. (2-11) [bar]</th>
<th>( V_{\text{consumed}}^a ) [nℓ/inspiration]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>15</td>
<td>4</td>
<td>1.867</td>
<td>1.133</td>
<td>1.133 x 1.867 = 2.115</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>1.867</td>
<td>3.000</td>
<td>3 x 1.867 = 5.601</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Totaal: 57.87 ℓ</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>5</td>
<td>2.333</td>
<td>1.167</td>
<td>1.167 x 2.333 = 2.723</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>2.333</td>
<td>3.000</td>
<td>3 x 2.333 = 6.999</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Totaal: 58.33 ℓ</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>6</td>
<td>2.800</td>
<td>1.200</td>
<td>1.200 x 2.800 = 3.360</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>2.800</td>
<td>3.000</td>
<td>3 x 2.800 = 8.400</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Totaal: 58.80 ℓ</td>
</tr>
</tbody>
</table>

* Exclusive the amount of air, necssary to keep the volume \( V_{\text{tot}} \) constant.

* Note that as the pressure changes linear with depth the total consumed air can be calculated as the mean value of the first and last breath mutiplied by \( v_b \).
3 ANALYSES
3.1 Air consumption during diving until ascend

The amount of air a diver consumes during descending is determined primarily by the required tidal volume (V_{work}) and although to a lesser degree, by keeping the total pulmonary volume constant/unchanged.

**Required amount of air to keep the total pulmonary volume constant during descending**

At the breathing moment i, the ambient pressure has changed from P_{i-1} to P_i; see fig. 01. For the total pulmonary volume, V_{tot} the corresponding nℓ volumes are obtained by:

- at t_{i-1} : P_{i-1} \cdot V_{tot, i-1} = P_0 \cdot [nℓ] (3-01-a)
- at t_i : P_i \cdot V_{tot} = P_0 \cdot [nℓ] (3-01-b)

The necessary air volume to keep V_{tot} constant at the breathing moment i is thus equal to:

\[ \delta V_{tot, i} = V_{tot, i} - V_{tot, i-1} \] (3-02)

Making use of the eqn. (2-09) and (3-01) yields, with P_0 ≡ 1 bar:

\[ \delta V_{tot, i} = v_{des, i-1} \cdot V_{work, i} \cdot 600^{-1} \] (3-03)

A reasonable average numerical value for the total lung capacity is (see table 01):

\[ V_{tot} \approx 6\frac{1}{2} \ell \] (3-03-a)

**Numerical example**

\[
\begin{align*}
V_{tot} & = 6\frac{1}{2} \ell \\
T_{resp} & = 4 \text{ s} \\
v_{des} & = 20 \text{ m/min} \\
D_{bottom} & = 20 \text{ m} \\
\delta V_{tot} & = 20 \times 4 \times 6\frac{1}{2} / 600 = 0.87 \text{ nℓ/pmoment of instantaneous breathing} \\
\Sigma \delta V_{tot} & = 0.87 \times (60 / 4) = 13 \text{ nℓ} \text{ at bottom depth} \\
\end{align*}
\]

The amount of air to keep the reference pulmonary volume constant during a descend to 20 m is rather small.

**Required air per breathing during descending**

The volume of air per act of breathing, \( \delta V_{work, i} \) in nℓ at a certain depth follows from:

\[ V_{work} \cdot P_i = P_0 \cdot \delta V_{work, i} \] (3-04)

With eq. (2-09) this turns into:

\[ \delta V_{work, i} = (1 + v_{des}[\text{m/min}] \cdot i \cdot T_{resp}[\text{s}] \cdot 600^{-1}) \cdot V_{work}[\text{ℓ}] \] (3-05)

**Numerical example (see earlier given data above also)**

Assume: \( V_{work} = 3 \ell \)

1\text{st} breathing i=1: \( \delta V_{work, 1} = (1 + 20 \times 1 \times 4 \cdot 600^{-1}) \times 3 = 3.4 \text{ nℓ} \)

15\text{th} breathing when \( D_{bottom} \) has been reached (i = 60/4): \( \delta V_{work, 15} = (1 + 20 \times 15 \times 4 \cdot 600^{-1}) \times 3 = 9 \text{ nℓ} \)

Summation over all breathings: \( \delta V_{work, 1 \rightarrow 15} = \Sigma i (1 + v_{des} \cdot i \cdot T_{resp} \cdot 600^{-1}) \cdot V_{work} = 93 \text{ nℓ} \)

**Air requirement during descending**

Combining the eqn. (3-03) and (3-05) yields the following result:

\[ \delta V_i = v_{des} \cdot T_{resp} \cdot V_{tot} \cdot 600^{-1} + (1 + v_{des} \cdot i \cdot T_{resp} \cdot 600^{-1}) \cdot V_{work}[\text{ℓ}] \] (3-06)

Or, if \( v_{des} \) and/or \( T_{resp} \) are variable and making use of the eqs. (2-06) and keeping in mind all quantities to identify with the right time index:

\[
\delta V_{a,i}[\text{nℓ/br.}] = \frac{RMV_{a,i}^{(1)}[\text{pℓ/\text{min}}] \cdot T_{resp,i-1} \cdot V_{tot} \cdot V_{des,i-1} + \sum_{j=0}^{i-1} \frac{RMV_{a,i}^{(1)}[\text{pℓ/\text{min}}]}{v_{b,j-1}} \sum_{j=0}^{i-1} v_{des,j} \cdot T_{resp,j}}{600} \] (3-07-a)

Alternatively:

\[
\delta V_{a,i}[\text{nℓ/br.}] = \frac{RMV_{a,i}^{(1)}[\text{pℓ/\text{min}}]}{v_{b,i-1}} + \left( V_{tot} \cdot V_{des,i-1} + \frac{RMV_{a,i}^{(1)}[\text{pℓ/\text{min}}]}{10 \cdot v_{b,i-1}} \sum_{j=0}^{i-1} \frac{v_{des,j}}{v_{b,j}} \right) \] (3-07-b)

\footnote{Or with the summation of an arithmetical series: \( \frac{1}{2} n (\text{term}_{\text{first}} + \text{term}_{\text{last}}) = \frac{1}{2} 15 (3.4 + 9) = 93 \).}
Note

i. It does not matter which of the two above given equations are used. It only matters that either \( T_{\text{resp}} \) or \( v_b \) is chosen as a parameter.

ii. The eqn. (3-07-a and -b) might give the impression that the total consumed air needed to dive to a certain depth, \( D_{\text{bottom}} \) is (strongly) dependent on \( T_{\text{resp}} \) resp. \( v_b \). However, this is not the case. For instance, assuming RMV, \( v_b \) and \( v_{\text{des}} \) are constant (i-independent) it follows from eq. (3-07-b) that:

\[
V_{a, \text{des}}[n\ell] = \frac{n}{10} \sum_{i=1}^{n} [\frac{\text{RMV}}{v_b} + \left( V_{\text{tot}} + \frac{\text{RMV}}{v_b} \right) \frac{v_{\text{des}}}{10v_b}] 
\]

(3-08-a)

in which: \( n \) [-] : number of inhalations until \( D_{\text{bottom}} \) has been reached

With \( n = v_b \cdot D_{\text{bottom}} \) / \( v_{\text{des}} \) the above given equation turns into:

\[
V_{a, \text{des}}[n\ell] = \frac{V_{\text{tot}}}{10} + \left( 1 + \frac{D_{\text{bottom}}}{20} + \frac{v_{\text{des}}}{20v_b} \right) \frac{\text{RMV}[p/ min]}{v_{\text{des}}} \left( V_{\text{tot}} \right) 
\]

(3-08-b)

For the numerical example given in paragraph 2.4 the same results are found by making use of eq. (3-08-b):

- for \( v_b \) is 15, 12 or 10 breathings/min \( V_{a, \text{des}} \) equals 57.87, 58.33 resp. 58.80 n\ell if \( V_{\text{tot}} \) is disregarded. This is in perfect agreement with the results given in table 03;

- the first term \( V_{\text{tot}} \cdot D_{\text{bottom}} /10 \) of 13 n\ell is the air supply to keep the total pulmonary volume unaffected. Though this is not much, it is still more than 20% of the total required air to reach the bottom at 20m.

Air consumption at bottom depth

\( \delta V_{a, \text{bottom}, i} = (1 + 0.1 \cdot D_{\text{bottom}}) \cdot V_{\text{work}}[p/ \ell] \) [n\ell /breathing i] (3-09-a)

or by making use of eq. (2-06):

\( \delta V_{a, \text{bottom}, i} = (1 + 0.1 \cdot D_{\text{bottom}} \cdot m) \frac{T_{\text{resp}, i-1}[\ell]}{60} \cdot \text{RMV} \cdot [p/ \ell] \) [n\ell /breathing i] (3-09-b)

3.2 Heat transfer from sea-water to the air in a dive bottle

In order to keep the process of heat transfer manageable, viz. not to complicate the problem unnecessarily it is assumed that the temperature distribution in sea-water, dive bottle and air therein is of a one-dimensional nature. Additionally, it is assumed also that the temperature of the dive bottle is homogeneous. This is shown schematically in the opposite figure.

Heat transfer from water to dive bottle:

\( dQ_1 = (T_w - T_b) \cdot h_{w-b} \cdot F_b \cdot \text{dt} \)

(3-10)

Heat transfer from dive bottle to air therein:

\( dQ_2 = (T_b - T_a) \cdot h_{b-a} \cdot F_a \cdot \text{dt} \)

(3-11)

in which: \( dQ/\text{dt} \) [J/ s]: heat transport per unit of time

\( h \) [W/ m² K]: heat transfer coefficient

\( F \) [m²]: heat transferring surface

\( T \) [°C]: temperature

'Heating up' of dive bottle:

\( dQ_1 = m_b \cdot c_b \cdot dT_b \)

(3-13)

'Heating up' of dive air:

\( dQ_2 = m_a \cdot c_a \cdot dT_a \)

(3-14)

in which:

\( m \) [kg]: mass

\( c \) [J/(kg °C)]: heat capacity per unit of mass

Elimination of \( dQ_1 \) and \( dQ_2 \) from the eqn. (3-10), (3-11), (3-13) and (3-14) yields the following set of common simultaneous differential equations:

\( dT_a/\text{dt} = a_a (T_w - T_a) \)

(3-16-a)

\( dT_b/\text{dt} = a_b (T_w - T_b) - \lambda a_a (T_b - T_a) \)

(3-16-b)
in which:

\[ a_a = (h_{ba} \cdot F_a) / (m_a \cdot c_a) \quad [1/s] \quad \text{thermal decay 'constant' of air in the dive bottle} \quad (3-17) \]

\[ a_b = (h_{wb} \cdot F_b) / (m_b \cdot c_b) \quad [1/s] \quad \text{thermal decay 'constant' of dive bottle} \quad (3-18) \]

\[ \lambda = (m_a \cdot c_a) / (m_b \cdot c_b) \quad [-] \quad \text{ratio of heat capacity of dive air and of dive bottle} \quad (3-19) \]

The solution of the set of differential equations (3-16) between \( t_{i-1} \) and \( t_i \) (thus for time index \( i \)) yields after some elementary analytical manipulations the following result:

\[
T_{a,i(-)} = T_w + C_{1,i} \cdot 2^{-\text{Resp}.i-1/t_1} + C_{2,i} \cdot 2^{-\text{Resp}.i-1/t_2} \quad \text{(see eq. (3-41), also)} \quad (3-20)
\]

\[
T_{b,i} = T_w + C_{1,i} \cdot F_1 \cdot 2^{-\text{Resp}.i-1/t_1} + C_{2,i} \cdot F_2 \cdot 2^{-\text{Resp}.i-1/t_2} \quad \text{(see eq. (3-42), also)} \quad (3-21)
\]

in which:

\[
C_{1,i} = \left( \frac{T_{a,i-1(+)} - T_w}{F_3} \right) - \left( \frac{T_{b,i-1} - T_w}{F_3} \right) \quad [^0\text{C}] \quad \text{('integration constant')} \quad (3-22)
\]

\[
C_{2,i} = T_{a,i-1(+)} - T_w - C_{1,i} \quad [^0\text{C}] \quad \text{('integration constant')} \quad (3-23)
\]

\[
\tau_a = \frac{\ln 2}{a_a} \quad [\text{s}] \quad \text{("half-value time of dive air")} \quad (3-24-a)
\]

\[
\tau_b = \frac{\ln 2}{a_b} \quad [\text{s}] \quad \text{("half-value time of dive bottle")} \quad (3-24-b)
\]

\[
A = \frac{1}{\tau_b} + \frac{1 + \lambda}{\tau_a} \quad [1/\text{s}] \quad B = \frac{1}{\tau_b \cdot \tau_a} \quad [1/\text{s}^2] \quad (3-25)
\]

\[
\tau_1 = \frac{2}{A - \sqrt{A^2 - 4B}} \quad [\text{s}] \quad \tau_2 = \frac{2}{A + \sqrt{A^2 - 4B}} \quad [\text{s}] \quad (3-26)
\]

\[
F_1 = 1 - \frac{\tau_a}{\tau_1} \quad [-] \quad F_2 = 1 - \frac{\tau_a}{\tau_2} \quad [-] \quad \left\{ \begin{array}{l}
F_3 = \frac{1}{\tau_1} - \frac{1}{\tau_2} \\
\tau_a = F_2 - F_1 \quad [-]
\end{array} \right. \quad (3-27)
\]

Note

The 'constants' \( C_{1,i} \) and \( C_{2,i} \) do change after each instantaneous inspiration \( i \).

At \( i = 0 \) it applies that: \( T_{a,(0)(+)} = T_a(t=0) \); see also calculation scheme fig. 05.

The progress of dive air temperature as well as that of the dive bottle can be calculated during each period of breathing by means of the set of equation (3-20) up to (3-27), except that in the set of integration constants eqn. (3-22) and (3-23) still one unknown quantity appears, viz. \( T_{a,i-1(+)} \). The value of which follows from the temperature drop on each moment of instantaneous inspiration, brought about through adiabatic expansion of the dive air in the dive bottle; see fig. 01 sub 2. As \( T_{a,i-1(+)} \) is known from the former breathing cycle 'i-1', \( T_{a,i-1(+)} \) follows from:

\[
T_{a,i-1(+)} = T_{a,i-1(-)} + \Delta T_{a,i-1} \quad (3-28)
\]

in which: \( \Delta T_{a,i-1} > 0 \) \([^0\text{C} \text{ or K}]\): temperature drop, due to adiabatic air expansion

Note

Coming back to the assumption that the dive bottle temperature is of a homogeneous nature, in annex A it is estimated to what extent this assumption can partially be relieved. It concerns the heat transfer through the dive bottle wall from sea-water to the air contained in the dive bottle. All eqs. (eq. 3-17) up to (3-27) inclusive remain valid except that \( a_a \) and \( \lambda \) have to be replaced by \( a_{a, \text{ref}} \) resp. \( \lambda_{\text{ref}} \); see the set of eqs. (A-07) and (A-08). Obviously, this adaptation stands for a steady state heat transfer process from sea-water to air in the dive bottle and does not include the effect on \( t_b \).

The numerical implication looks as follows. With the base line data in annex B and in anticipation of the determination of the nominal heat transfer coefficients resulting in \( h_{wb} \approx 700 \text{ W/m}^2 \text{ K} \) and

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h_{ba} \approx 50 \text{ W/m}^2 \text{ K} the following k_{\text{effect}}-values are obtained:

\[
k_{\text{effect \_ waterside}} = \frac{h_{w-b}}{k_b} \cdot t_b \cdot \frac{F_b}{F_a + F_b} = 700 \cdot 0.0149 \cdot 0.417 / \{175 \cdot (0.417 + 0.324)\} = 3.35 \%
\]

\[
k_{\text{effect \_ air side}} = \frac{h_{b-a}}{k_b} \cdot t_b \cdot \frac{F_a}{F_a + F_b} = 50 \cdot 0.0149 \cdot 0.324 / \{175 \cdot (0.417 + 0.324)\} = 0.19 \%
\]

Obviously, though the adaptation deals only partially with the heat conduction in the wall of the dive bottle (does not effect \(a_b\)), the assumption that the temperature of the dive bottle is homogeneous has no significant repercussions. In anyway the corresponding errors in the calculation results are certainly much smaller than the uncertainties in the adopted heat transfer coefficients, \(h_{w-b}\) and \(h_{b-a}\).

### 3.3 Estimation of air temperature drop brought about through instantaneous inspiration.

In case of adiabatic expansion - a process in which case neither heat is supplied nor subtracted - it follows that on the moment in time of \(t_i\), see for instance [02] page A4/19:

\[
\frac{T_{A,i(-)}}{T_{A,i(+)}} = \left(\frac{P_{a,i(-)}}{P_{a,i(+)}}\right)^{\gamma-1} \frac{\gamma}{\gamma} \quad \text{(3-29)}
\]

in which:

- \(T_A\) [K] air temperature in the dive bottle; \(T_A[K] = 273.15 + T_a[°C]\)
- \(P_a\) [bar] air pressure in the dive bottle
- \(\gamma\) [-] ratio of specific heat of air at constant pressure, \(c_p\) and that at constant volume \(c_v\);
- \(\gamma = c_p / c_v\)

Applying the law of Boyle Gay-Lussac in an elementary fashion it turns out that the amount of inspired air at the moment in time \(t_i\) equals:

\[
\delta V_{a,i} = \left(\frac{P_{a,i(-)}}{T_{A,i(-)}} - \frac{P_{a,i(+)}}{T_{A,i(+)}}\right) \cdot \frac{T_{ref}}{P_{ref}} \cdot V_{db}
\]

in which:

- \(\delta V_{a,i}\) [\text{n}l] instantaneous inspired air volume at time index \(i\)
- \(T_{ref}\) [K] reference temperature
- \(P_{ref}\) [bar] reference pressure
- \(V_{db}\) [\text{l}] internal volume of dive bottle

On the moment in time \(t_i\) the quantities \(T_{A,i(-)}\) en \(P_{a,i(-)}\) are known. The unknown quantities \(T_{A,i(+)}\) en \(P_{a,i(+)}\) can than be calculated by means of the eqn. (3-29) and (3-33); two equations with two unknowns. After some mathematical manipulations, the following result is obtained to express \(\Delta T_{a,i}\)\(^2\) = \(T_{A,i(-)} - T_{A,i(+)}\) (eq. (3-28)) in known quantities:

\[
\Delta T_{a,i} = \left(1 - \left(1 - \frac{T_{A,i(-)}}{P_{a,i(-)} \cdot V_{db} \cdot T_{ref}}\right)^{\gamma-1}\right) \cdot \frac{T_{A,i(-)}}{P_{a,i(-)} \cdot V_{db} \cdot T_{ref}} \cdot (\gamma - 1) > 0 \quad \text{(3-37)}
\]

With \(P_{ref} = 1\) bar, \(T_{ref} = 273.15\) K, \(V_{db} = 12\) l and \(\gamma = 1.4\) eq. (3-37) turns in:

\[
\Delta T_{a,i} \approx 1.22 \left(\frac{T_{A,i(-)} / 100}{P_{a,i(-)}}\right)^2 \delta V_{a,i} \quad \text{(3-37-a)}
\]

Finally \(P_{a,i(+)}\) is obtained by combining eq. (3-29) and (3-37):

\footnote{Note that \(\Delta T_{a,i} > 0\) is by definition a temperature drop!}
Numerical example

Given: \( V_{db} = 12 \text{ ℓ} \), \( P_{ref.} = 1 \text{ bar}, T_{A,i(-)} = 305 \text{ K}, T_{ref.} = 273 \text{ K}, P_{a,i(-)} = 200 \text{ bar} \) and \( \gamma = 1.40 \).

Question: Estimate air temperature and pressure drop in the dive bottle for the former in paragraph 3.1 estimated inspired air volumes?

Result (values between parenthesis are approximations: \( \approx \) in concerned equations):

<table>
<thead>
<tr>
<th>Breath</th>
<th>( \delta V_a ) [nℓ]</th>
<th>( \Delta T_a ) [°C]</th>
<th>( \Delta P_a ) [bar]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st breath</td>
<td>3.4</td>
<td>+ 0.193 (+ 0.193)</td>
<td>- 0.443 (- 0.443)</td>
</tr>
<tr>
<td>15th breath</td>
<td>9.0</td>
<td>+ 0.512 (+ 0.511)</td>
<td>- 1.173 (- 1.173)</td>
</tr>
<tr>
<td>1st - 15th breath</td>
<td>93</td>
<td>+ 5.352 (+ 2.82)</td>
<td>- 12.02 (- 12.12)</td>
</tr>
</tbody>
</table>

3.4 Change in dive air pressure between breathings

*Between two successive* inspirations by definition (modelling), no air is subtracted from the dive bottle; see fig. 01. In other words, the pressure change in the dive bottle during the time lapse between two successive inspirations is of an isochoric nature and is equal to:

\[
P_{a,i(-)} = \frac{T_{A,i(-)}}{T_{A,i(+)}} \cdot P_{a,i(+)}
\]

Finally, by making use of the above given equation, the *decrease* in air pressure in the dive bottle at the commencement of an inspiration until the next inhalation is given by:

\[
\Delta P_{a,i}(\zeta) \equiv P_{a,i(-)} - P_{a,i(+)} = P_{a,i(-)} - \left( \frac{T_{A,i(\zeta)}}{T_{A,i(+)}} \right) \cdot P_{a,i(+)}, \quad 0 \leq \zeta \leq T_{resp,i}
\]

in which:

- \( T_A \) [K]
- \( \Delta P_a \) and \( P_a \) [bar]

3.5 Numerical solution procedure

The numerical solution of the heat transfer process is summarized in the calculation scheme below; calculation steps ① up to ⑥ inclusive.

---

**Fig. 05** Calculation scheme showing the computational steps, how the non-linear heat transfer process is solved.
Though the calculation procedure is rather straightforward, the effort to do so is very tedious. The reason is that the integration coefficients $C_{1,i}$ and $C_{2,i}$ (see the eqn. (3-22 resp. (3-23)) are not constant but time dependent. The driving parameters are $h_{w-b}$, $h_{b-a}$ and $m_a$ (see the eqn. (3-17), (3-18) resp. (3-19)). In other words, the time dependent values of $\tau_b$, $\tau_a$ and $\lambda$ have to be determined in order to proceed through the above given calculation steps up to inclusive. The estimation of these parameters is subject of the next chapter. For the computational results, see for:
- $\tau_b$: eqn. (4-03) and (4-13);
- $\tau_a$: eqn. (4-39) and (4-40);
- $\lambda$: eq. (4-42).

Note
The progress in air and bottle temperature in function of time between the time steps $t_{i-1}$ and $t_i$ is given by (see the eqs. (3-20) and (3-21)):

$$T_a(\zeta_i) = T_w + C_{1,i+i} \cdot 2^{-\zeta / \tau_1} + C_{2,i+i} \cdot 2^{-\zeta / \tau_2} \quad 0 \leq \zeta < T_{\text{resp.},i}$$

$$T_b(\zeta_i) = T_w + C_{1,i+i} \cdot F_1 \cdot 2^{-\zeta / \tau_1} + C_{2,i+i} \cdot F_2 \cdot 2^{-\zeta / \tau_2} \quad 0 \leq \zeta < T_{\text{resp.},i}$$

in which:

- $T_a(\zeta=0) = T_{a,i+1}$
- $T_a(\zeta=T_{\text{resp.},i}) = T_{a,i+1}$
- $T_b(\zeta=0) = T_{b,i}$
- $T_b(\zeta=T_{\text{resp.},i}) = T_{b,i+1}$
4 Justification of numerical data

4.1 Introduction
Calculation results obtained by means of even a general accepted model will not be more reliable/trustworthy than the used input data or other material constants. For that reason, this chapter deals with the following related aspects:
- specifications and schematization of dive bottle;
- dimensionless groups (numbers) in case of heat transfer phenomena;
- heat transfer coefficient between seawater and dive bottle;
- heat transfer coefficient between dive bottle and dive air therein;
- thermal decay 'constant' of dive air;
- ratio between heat capacity of dive air and dive bottle.

4.2 Dive bottle

Specifications
The author was asked to estimate the temperature of the air contained within a dive bottle during diving. To that end the following preliminary data of the dive bottle was supplied:
- material: Al (assumed)
- \( m_b = 15 \) kg
- \( V_{db} = 12 \) ℓ (internal volume)
- \( H_b = 60 \) cm

Data obtained from internet www.techdivinglimited.com (Technical Diving Limited, TDL 'a leading seller') learns that there are all-in-all 26 Al and 60 steel dive bottles of different sizes and manufacturers. For an aluminium or steel bottle with \( V_{db} \approx 12 \) ℓ three (Luxfer 92, Luxfer 100 en Catalina C100) resp. more than 10 items were scrutinized. However, the mass of the aluminium dive bottles were much larger than the above-mentioned value of 15 kg, viz.: 17.1, 18.5 resp. 20.9 kg. Being asked which bottle it concerns no more addition data could be supplied, otherwise than that it most probably concerns a Luxfer dive bottle. In this connection it is noted that specifications supplied by Luxfer itself, www.luxfercylinders.com/... do not comply with those of TDL and that Luxfer does not mentioned a type identification; see table 04.

Table 04 Characteristics of some of the aluminium Luxfer dive bottles

<table>
<thead>
<tr>
<th></th>
<th>Luxfer 92</th>
<th>Luxfer 100</th>
<th>Luxfer 'site'</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{db} )</td>
<td>11.6</td>
<td>12.3</td>
<td>12.0</td>
</tr>
<tr>
<td>( d_{ext} )</td>
<td>2.03</td>
<td>2.03</td>
<td>2.04</td>
</tr>
<tr>
<td>( P_{service} )</td>
<td>221</td>
<td>228</td>
<td>200 (197.4 st. atm.)</td>
</tr>
<tr>
<td>( m_b )</td>
<td>17.1</td>
<td>18.5</td>
<td>16.5</td>
</tr>
<tr>
<td>( H )</td>
<td>6.30</td>
<td>6.66</td>
<td>6.12</td>
</tr>
<tr>
<td>( B_{empty} )</td>
<td>1.41</td>
<td>1.41</td>
<td>not mentioned; + 1.40 kg</td>
</tr>
<tr>
<td>( B_{full} )</td>
<td>-1.63</td>
<td>-1.97</td>
<td>not mentioned; - 1.48 kg</td>
</tr>
</tbody>
</table>

1) With the mentioned buoyancy forces and with \( \rho_{air, 1 \text{ bar}, 20 \, \text{C}} = 1.20 \times 10^{-3} \) kg/dm\(^3\) it is found that \( V_{db} \) equals: 

\[
V_{db} = \frac{B_{full} - B_{empty}}{(\rho_{air, 1 \text{ bar}} \cdot P_{service})} = 11.5 \ell. 
\]

This is obviously in good agreement with the value mentioned in the table.

2) As above: \( V_{db} = 12.4 \ell \). Again a good agreement with the value mentioned in the table.

3) Empty weight without valve.

4) Probably also empty weight without valve.

5) Without valve.

6) Probably also without valve.

7) \( B \) stands for buoyancy force.

8) Calculated with: \( B_{empty} = V_{11} \cdot \rho_{\text{sea water}} \cdot m_b - V_{db} \cdot \rho_{air, 1 \text{ bar}} \).

9) Calculated with: \( B_{full} = B_{empty} - V_{db} \cdot \rho_{air, 1 \text{ bar}} \cdot P_{service} \).

In concluding, the above-mentioned preliminary specifications of the dive bottle turn out to be

---

10 End of 2013.
11 Calculated with eq. (4-01-c).
incorrect. Therefore, all further considerations/calculations are based on the data compiled in the last column of table 04.

**Schematization**

The dive bottle is schematized as a cylinder with outer diameter \(d_{\text{ext}}\), a wall thickness \(t\) and an effective height \(H_{\text{eff}}\). Assuming that the upper and bottom side of the schematized dive bottle are flat with a thickness of \(1\frac{1}{2}t\) the external volume of the dive bottle \(V_b\) and the contained 'water volume' \(V_{db}\) can be expressed as:

\[
V_b = \frac{1}{4} \pi d_{\text{ext}}^2 H_{\text{eff}}
\]

\[
V_{db} = \frac{1}{4} \pi (d_{\text{ext}} - 2t)^2 (H_{\text{eff}} - 3t)
\]

With \(m_b = (V_b - V_{db}) \cdot \rho_{\text{Al}}\) it is found that:

\[
m_b = \{1 - (1 - 2t/d_{\text{ext}})^2 \cdot (1 - 3t/H_{\text{eff}})\} \frac{1}{4} \pi d_{\text{ext}}^2 H_{\text{eff}} \cdot \rho_{\text{Al}}
\]

For the aluminium dive bottle, Luxfer 'site' with \(d_{\text{ext}} = 2.04\ \text{dm}, \ V_{db} = 12\ \text{ℓ}\), \(m_b = 16.5\ \text{kg}\) and \(\rho_{\text{Al}} = 2.78\ \text{kg/dm}^3\) the unknowns \(t\) and \(H_{\text{eff}}\) computed as the solution of the eqn. (4-01-b) and (4-02); graphically determined - by eliminating \(H_{\text{eff}}\) from eq. (4-02) by means of eq. (4-01-b) - as the intersection point of \(m_b(t)\) with \(m_b(t) = 16.5\ \text{kg}: \ t \approx 14.9\ \text{mm}\) and \(H_{\text{eff}} \approx 5.49\ \text{dm}\)

**Note**

\(H_{\text{eff}} (= 5.49\ \text{dm})\) is of course smaller than \(H (= 6.12\ \text{dm})\) due to the tapering of from cylinder to valve threat.

With these data, the heat transferring areas and \(V_b\) can be determined:

\[
F_b = (H_{\text{eff}} + ½ d_{\text{ext}}) \pi d_{\text{ext}} = 41.7\ \text{dm}^2 = 0.417\ \text{m}^2
\]

\[
F_a = \{(H_{\text{eff}} - 3t) + ½ (d_{\text{ext}} - 2t)\} \pi (d_{\text{ext}} - 2t) = 32.4\ \text{dm}^2 = 0.324\ \text{m}^2
\]

\[
V_b \approx 17.94\ \text{ℓ}
\]

With \(c_{b,\text{Al}} = 880\ [\text{J/kg}\cdot\text{K}]\) and the eqn. (3.18) and (3-19) the half-value time of the dive bottle equals:

\[
\tau_b[s] = \frac{m_b \cdot c_{b,\text{Al}}}{F_b \cdot h_{w-b}} \cdot \ln 2 = 16.5 \times 880 \times \ln 2 / (0.417 \times h_{w-b}) = 24.1\ 10^3/J\cdot\text{m}^2\text{K}
\]

4.3 Dimensionless groups (numbers) in case of heat transfer phenomena

Heat transfer coefficients are 'hidden' within the so called dimensionless characteristic numbers, see for instance [02] page. A4/26:

- Nusselt number:
  \( \text{Nu} = \frac{h \cdot L}{k} \)  \([-]\)  (4-04)

- Prandtl number:
  \( \text{Pr} = \frac{\mu \cdot c_p}{k} \)  \([-]\)  (4-05)

- Reynolds number:
  \( \text{Re} = \frac{\rho \cdot V \cdot L}{\mu} \)  \([-]\)  (4-06)

- Grashof number:
  \( \text{Gr} = \frac{L^3 \cdot g \cdot \beta \cdot \Delta T}{\nu^2} \)  \([-]\)  (4-07)

in which:

- \(h\) \[W/(m^2 \cdot \text{K})\] heat transfer coefficient
- \(L\) \[m\] characteristic length
- \(k\) \[W/(m \cdot \text{K})\] heat conduction coefficient
- \(\mu\) \[Pa.s\] dynamic viscosity (1 Pa \(\equiv 1\ N/m^2\))
- \(c_p\) \[J/(m^3 \cdot \text{K})\] specific heat at constant pressure
- \(\rho\) \[kg/m^3\] specific mass
- \(V\) \[m/s\] velocity
- \(g\) \[m/s^2\] gravitational acceleration
- \(\beta\) \[m^3/(m^3 \cdot \text{K})\] volumetric thermal expansion coefficient
- \(\Delta T\) \[\text{K}\] characteristic temperature difference
- \(\nu\) \[m^2/s\] kinematic viscosity (\(\nu \equiv \mu/\rho\))
4.4 Heat transfer coefficient between seawater and dive bottle

**Introduction**

In the case at hand, the determination of the numerical value of the heat transfer coefficient is accompanied with a large margin of uncertainty. The reason therefore is that the dive bottle is located in the wake of the diver, is partly 'wrapped up' by the bottle-carrier set and, moreover the dive bottle can be inclined in any position. A large margin in the heat transfer coefficient must thus be taken into account, or if not acceptable than the heat transfer coefficient should be established experimentally, which is a prolonged and possibly costly effort.

**First estimation of \( h_{wb} \)**

From \([02]\) - first column beneath in table 05 beneath - and \([08]\) - second column - the following indicative values for \( h_{wb} \) are compiled.

<table>
<thead>
<tr>
<th></th>
<th>( \text{gas, natural convection} )</th>
<th>( \text{gas, forced convection} )</th>
<th>( \text{fluid, natural convection} )</th>
<th>( \text{fluid, forced convection} )</th>
<th>( \text{water, natural convection} )</th>
<th>( \text{water, forced convection} )</th>
<th>( \text{conv., with phase transition} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 - 15</td>
<td>10 - 100</td>
<td>50 - 1000</td>
<td>500 - 3000</td>
<td>350 - 580</td>
<td>580 - 2300</td>
<td>2,500 - 100,000</td>
</tr>
</tbody>
</table>

**Numerical results**

Assume on basis of the above mentioned data: \( h_{wb} \approx 600 \text{ W/(m}^2\text{K)} \);

\[ \tau_b = 40.2 \text{ s} = \frac{2}{3} \text{ min.} \]

During discussions within the board of 'Stichting Duik Research' a value of 2.5 min was mentioned. Or in other words \( h_{wb} \) should be 80 \text{ W/(m}^2\text{K)}; a factor eight lower which seems not to be realistic.

In order to get some feeling for the influence of the air mass contained within the dive bottle at maximum service pressure (200 bar) the heat capacity of the dive bottle is augmented with that of the contained air. With \( m_a = 2.85 \text{ kg} \) and other data given in annex B, eq. (4-03) turns in (at \( T_a = 200 \text{ C} \)):

\[ \tau_{b+a} = \frac{m_b \cdot c_b \cdot A_b + m_a \cdot c_a \cdot V}{F_b \cdot h_{wb}} \ln 2 = 47 \text{ s} \approx \frac{3}{4} \text{ min} \]

The half-value time is larger than that of an empty dive bottle as expected, however only for 15 %.

**Further assessment of \( h_{wb} \) with forced convection**

There is an extensive amount of literature available concerning heat transfer through forced cross-sectional flow around a cylinder and a sphere; see for instance \([03]\) en \([04]\). Heat transfer data in case of external axial flow around a cylinder however is not found to be present in these references. However, it is for a flow along a flat plate. On physical grounds it is for certain to state that heat transfer predictions along a flat plate result in an underestimation of that in case of external axial flow along a cylinder.

Ref. \([03]\) page 498 and 499

**Laminar horizontal flow along a flat plate** (one-sided heat transport):

\[ \text{Nu}_x = 0.664 \cdot \text{Re}^{0.9} \cdot \text{Pr}^{1/3} \text{ for a laminar flow:} \quad \text{Re}_x < 300.000 \quad (4-09) \]

in which: \( x \) [m] distance to the beginning of the plate; \( 0 < x \leq H_{\text{eff}} \).

\[ \begin{align*}
\text{Pr}_{\text{seawater}} & = 6.62 \text{ with:} \\
\mu_{\text{seawater}} & = 1.01 \cdot 10^{-3} \text{ Pa.s} \\
C_p, \text{ seawater} & = 3.93 \cdot 10^3 \text{ J/(kg}^3\text{.K}) \\
k_{\text{seawater}} & = 0.6 \text{ W/(m} \cdot \text{K)} \\
\text{Re}_x, \text{ seawater} & = Vx/v \\
V_{\text{seawater}} & = \mu_{\text{seawater}} / \rho_{\text{seawater}} \\
\rho_{\text{seawater}, 3\% \text{ salt}} & = 1.024 \text{ kg/m}^3 \\
V_{\text{seawater}} & = 0.986 \cdot 10^6 \text{ m}^3/\text{s} \\
v & = 20 \text{ m/min} = \frac{1}{3} \text{ m/s} \\
\end{align*} \]
\[ \text{Re}_{x, \text{seawater}}^{12} = 338,000 \cdot x \quad \text{x} = L_{\text{plate}} \quad \sim H_{\text{eff}} = 0.549 \, \text{m (see under paragraph 4.2)} \]
\[ \text{Re}_{x} \quad \sim 186,000 < 300,000 \]

Apparently the flow is laminar.

\[ \text{Nu}_{x} = h_{w-b} \cdot \text{x} / k \]

Substitution of the above given numerical values of the numbers of Pr, \( \text{Re}_{x} \) and \( \text{Nu}_{x} \) in eq. (4-09) yields:
\[ h_{w-b, \text{ laminar}} = 0.664 \cdot 338,000^{1/2} \cdot 6.62^{1/3} \cdot 0.6 \cdot x^{-1/2} = 435 \cdot x^{-1/2} = 435/\sqrt{0.549} = 587 \, \text{W/m}^{2} \cdot \text{K.} \]

Assuming more or less arbitrarily that the undisturbed heat transfer is effective for \( \frac{3}{4} \) approximately - for reasons of disturbing wake effects and that of the bottle-carrier set - then the following results are obtained:
- \( h_{w-b, \text{ laminar}} \mid V=20 \, \text{m/min} \approx 440 \, \text{W/(m}^{2} \cdot \text{K)} \)
- eq. (4-03) \( \rightarrow t_{b} = 55 \, \text{s} = 0.9 \, \text{min} \)

Note

Compared with the value of 2.5 min for \( t_{b} \) mentioned during discussions within the board of ‘Stichting Duik Research’ the difference is slightly larger than a factor 2.5, disregarding for the moment the presence of air in the dive bottle.

To obtain some feeling for the fact that the flow along the dive bottle might be more likely of a turbulent than of a laminar nature due to the former mentioned disturbances the following alternative result comes into view.

**Turbulent horizontal flow along a flat plate** (onesided heat transport)\(^{13}\):

\[ \text{Nu}_{x} = 0.0325 \cdot \text{Re}_{x}^{0.8} \cdot \text{Pr}^{1/3} \]

With earlier mentioned numerical data, the following result is obtained:
\[ h_{w-b, \text{ turbulent}} = 0.0325 \cdot 338,000^{0.8} \cdot 6.62^{1/3} \cdot 0.6 \cdot x^{-0.2} = 970 \cdot x^{-0.2} = 970 \cdot 0.549^{-0.2} = 1.094 \, \text{W/m}^{2} \cdot \text{K.} \]

Thus if so assumed as in case of a laminar flow, effective for 75%:
\[ h_{w-b, \text{ turb}} \mid V=20 \, \text{m/min} = 820 \, \text{W/m}^{2} \cdot \text{K} \]

This value is about 1.9 times larger that in case of a laminar flow!

**Ref. [08] page 385**

For an undisturbed flow along a flat plate the ideal state of flow the initial laminar flow will turn into a turbulent flow if the plate length \( L_{\text{plate}} \) will be large enough. The transition point \( x_{\text{crit.}} \) is determined by \( \text{Re}_{\text{crit.}} \):
\[ \text{Re}_{\text{crit.}} = \rho \cdot V \cdot x_{\text{crit.}} \cdot \mu \]
\[ = 10^{5} \text{ to } 5 \cdot 10^{6} \]
\[ \text{Re}_{\text{crit.}} \{\text{assumed base line value} \} = 5 \cdot 10^{5} \]

The minimum value of \( x_{\text{crit.}} \) occurs at the maximum value of \( V \) (\( \equiv V_{d} = v_{\text{des.}} / \cos \alpha \)), being 35 m/min see eq. (2-05). Thus with \( \mu_{\text{seawater}} = 1.01 \cdot 10^{-3} \) Pa.s and \( \rho = 1.024 \cdot 10^{3} \) kg/m\(^{3}\):
\[ x_{\text{crit.}} \geq 5 \cdot 10^{3} \cdot \mu / (\rho \cdot V) = 0.845 \, \text{m} \]

In other words, theoretically the flow along the entire plate length will be laminar as \( H_{\text{eff}} \) equals 0.549 m, which is in agreement with the above extracted data from ref. [03].

**Adopted parametric values of \( h_{w-b} \)**

Though formally speaking the flow around the dive bottle will be probably laminar the first pressure reduction valve, isolation valve, pressure tube, carrier set of the dive bottle and wake effects are serious disturbing effects to provoke a turbulent flow.

\(^{12}\) \( x \) must not be interpreted as at the local position \( x \), so as mentioned in [03], but as the plate length.

See [08] page 423 in particular.

\(^{13}\) Integration of [08] eq. (7.30) on page 424 results into a value of 0.0370 in stead of 0.0325 in eq. (4-10).
Depending on \(v_d\) (swimming velocity of diver) and assuming that the 'theoretical' heat transfer coefficients are effective for only 75\% the eqs. (4-09) and (4-10) can be rewritten as:

\[
\begin{align*}
\text{h}_{w-b, \text{ laminar}} &= 763 \cdot \sqrt{v_d \, \text{[m/s]}} \\
\text{h}_{w-b, \text{ turbulent}} &= 1976 \cdot (v_d \, \text{[m/s]})^{0.8}
\end{align*}
\]  

(4-11) (4-12)

By considering all uncertainties, the author's best guess for a realistic calculation rule of the heat transfer coefficient between seawater and dive bottle is:

\[
\text{h}_{w-b} \, \text{[W/m}^2\text{. K]} = 1250^{\pm 400} \cdot (v_d \, \text{[m/s]})^{0.6} \quad 16 \leq v_d \leq 35 \, \text{m/min}
\]  

(4-13)

The practical implications of the relative large spread in \(h_{w-b}\) - and thus \(\tau_b\) see eq. (4-03) - as a parameter. In case \(v_d = 23.1\) m/min\(^{15}\), therefore the following numerical results will be used:

\[
\text{h}_{w-b|v_d = 23.1 \, \text{m/min}}: 480, 700 \quad \text{resp.} \quad 930 \text{ W/m}^2\text{K}, \quad \text{corresponding with} \quad \tau_b: 50, 34 \quad \text{resp.} \quad 26 \, \text{s.}
\]

**Numerical example and justification**

These calculation values of \(h_{w-b}\) do agree satisfactorily with the baseline values mentioned in table 05, ranging from 580 to 2300 W/(m\(^2\)K).

Table **06** Heat transfer coefficient of seawater-dive bottle in function of swimming velocity

<table>
<thead>
<tr>
<th>(v_d) [m/min]</th>
<th>(\text{h}_{w-b, \text{ laminar}}), (\text{eq. (4-11)})</th>
<th>(\text{h}_{w-b, \text{ turbulent}}), (\text{eq. (4-12)})</th>
<th>(\text{h}_{w-b, \text{ min.}}), (\text{eq. (4-13)})</th>
<th>(\text{h}_{w-b, \text{ best estimate}}), (\text{eq. (4-13)})</th>
<th>(\text{h}_{w-b, \text{ max.}}), (\text{eq. (4-13)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>394</td>
<td>686</td>
<td>385</td>
<td>566</td>
<td>747</td>
</tr>
<tr>
<td>20</td>
<td>441</td>
<td>821</td>
<td>440</td>
<td>647</td>
<td>854</td>
</tr>
<tr>
<td>23.1</td>
<td><strong>473</strong></td>
<td><strong>921</strong></td>
<td><strong>479</strong></td>
<td>(\rightarrow 705 \quad \leftarrow)</td>
<td><strong>931</strong></td>
</tr>
<tr>
<td>24</td>
<td>483</td>
<td>949</td>
<td>491</td>
<td>721</td>
<td>952</td>
</tr>
<tr>
<td>28</td>
<td>521</td>
<td>1074</td>
<td>538</td>
<td>791</td>
<td>1044</td>
</tr>
<tr>
<td>32</td>
<td>557</td>
<td>1195</td>
<td>583</td>
<td>857</td>
<td>1132</td>
</tr>
<tr>
<td>35</td>
<td>583</td>
<td>1284</td>
<td>615</td>
<td>905</td>
<td>1194</td>
</tr>
</tbody>
</table>

**4.5 Heat transfer from dive bottle to dive air contained therein**

**Introduction**

Estimation of heat transfer from the dive bottle to the air contained therein and vice versa is cursed with a still larger uncertainty margin than that of seawater to dive bottle. In the case at hand, heat transfer only takes place through natural convection within the dive bottle. No heat is transferred as radiation heat; see [03] page 504.

In accordance with eq. (3-11) the heat transfer coefficient \(h_{b-a}\) is defined as:

\[
q_2 = h_{b-a} \cdot (T_b - T_a)
\]  

(4-14)

in which: \(q_2\) [J/m\(^2\).s] heat density rate from dive bottle to the air contained therein

**Heat transfer correlations for \(h_{b-a}\)**

Ref. [08]

The following heat transfer correlation for free convection along an inclined vertical plate is extracted from [08] eq. (9.27):

\[
\begin{align*}
\text{Nu}_{\text{lam.}} &\approx 0.67 \cdot \left\{ 1 + f(\text{Pr}) \cdot \text{Gr}^{0.34} \right\} & \text{all \ material \ constants \ at \ P_a \ en \ T_{\text{film}}} \\
\text{f(Pr)} &= \text{Pr}^{0.1} \left\{ 1 + 0.492 / \text{Pr}^{0.16} \right\} \cdot 4^{0.16} \\
\text{T}_{\text{film}} &= \text{def.} \frac{1}{2} (T_b + T_a)
\end{align*}
\]  

(4-15) (4-16) (4-17)

in which:

\[
\begin{align*}
\text{Nu} &= [-] \quad \text{number \ of \ Nusselt, \ see \ eq. \ (4-04)}
\end{align*}
\]

\(^{14}\) Laminar/turbulent flow with: \(\text{Pr}_{\text{seawater}} = 6.62, \ 	ext{Re}_{x, \text{seawater}} = 556,700 \cdot v_d\, \text{[m/s]}\) and \(\text{h}_{w-b,laminar}[\text{W/m}^2\text{K}]= 0.75 \cdot 1.093 \text{Nu}_{\text{lamina}}\)

\(^{15}\) For \(v_{\text{des}} = 20 \, \text{m/min and} \ a = 30°\).
Gr [-] number of Grashof, see eq. (4-07), and with:
\[
\Delta T \text{ def. } T_b - T_a
\]
g \rightarrow g \cos \alpha

Pr [-] number of Prandtl, see eq. (4-05)

P_a [bar] air pressure in dive bottle

T_{film} [K] so called 'film' temperature

Ref. [04]
The following heat transfer correlations for free convection along a vertical plate are obtained from [04] page 172 and 173:

\[
\text{Nu}_{sub \ lam} 1) \approx 1 + 0.50 \cdot (Pr \cdot Gr)^{1/4} \quad \text{all material constants at P_a en T-film}
\]

\[
\text{Nu}_{lam} 2) \approx 0.59 \cdot (Pr \cdot Gr)^{1/4} \quad \text{all material constants at P_a en T-film}
\]

\[
\text{Nu}_{turbulent} 3) \approx 0.13 \cdot (Pr \cdot Gr)^{1/3} \quad \text{all material constants at P_a en T-film}
\]

1) Analytical approach of numerical data for air so as mentioned in [04].

Gr , Pr Nu [04] fig. 7-7 Nu eq. (4-20-a)
10^6 1.44 1.50
10^7 1.90 1.90
10^8 2.63 2.58
10^9 3.89 3.81
10^10 6.03 6.00

2) [04] eq. (7-4b)

3) [04] eq. (7-4a)

Further elaborations on the above given heat transfer correlations will be dealt with further down in this paragraph.

**Pressure and temperature dependent thermal-physical properties of air**

For the thermal-physical properties of air at a pressure P_a and temperature T_{film} the following expressions are employed.

**Specific mass of air, \( \rho_a \)**

By applying the law of Boyle-Gay Lussac for an ideal gas it is found that:

\[
\rho_a = \rho_{a, \text{ref.}} \cdot \left( \frac{P_{P_{\text{ref.}}}}{P_a} \right) \cdot \left( \frac{T_{\text{ref.}}}{T_a} \right)
\]

With \( \rho_{a, \text{ref.}} = 1.29 \text{ kg/m}^3 \) extracted from [02][16] at 1 atm (\( P_{a, \text{ref.}} = 1.013 \text{ bar} \)) and 273.15 K (= \( T_{a, \text{ref.}} \)) and \( T_a = T_{film} \)[17] the expression for \( \rho_a \) turns into:

\[
\rho_a [\text{kg/m}^3] = 348 \cdot \frac{P_a [\text{bar}]}{T_{film} [\text{K}]} \quad \left( T_{film} [\text{K}] \text{ or } T_a [\text{K}] \right)
\]

**Volumetric thermal expansion coefficient of air, \( \beta_a \)**

Again applying the law of Boyle-Gay Lussac for an ideal gas it appears that:

\[
\beta_a [\text{K}^{-1}]|_{P=\text{const.}} = \left( T_{film} [\text{K}] \right)^{-1}
\]

**Dynamic viscosity of air, \( \mu_a \)**

The dynamic viscosity of air is independent of the prevailing pressure P_a but it is from temperature. For an ideal gas the relation of Sutherland[18] applies:

\[
\mu_a = \mu_a (T_{\text{ref.}}) \cdot \frac{T_{\text{ref.}} + C_a}{T_{film} + C_a} \left( \frac{T_{film}}{T_{\text{ref.}}} \right)^{3/2}
\]

in which: \( C_a [\text{K}] \) nondimensional constant of Sutherland; for air: \( C_a = 120 \text{ K} \)

With \( \mu_a = 18.27 \cdot 10^{-6} \text{ Pa.s} \) at \( T_{\text{ref.}} = 291.15 \text{ K} \) the above given equation turns in:

\[
\mu_a [\text{Pa.s}] = 1.51 \cdot 10^{-6} \cdot \frac{\sqrt{T_{film} [\text{K}]}}{1 + 120/T_{film} [\text{K}]}
\]

---


[17] If it concerns natural heat transfer phenomena, otherwise \( T_a \).

Eq. (4-23-b) can be simplified at the expense of an error smaller than $\frac{1}{2}$ %:

\[
\mu_a [\text{Pa.s}] \approx 0.195 \times 10^6 \cdot (T_{\text{film}}[\text{K}])^{0.80} \quad 250 \text{ K} \leq T_{\text{film}} \leq 350 \text{ K} \quad (4-23-c)
\]

**Heat conduction coefficient of air, \( k_a \)**

With data extracted from [08] the following empirical relation for \( k_a \) can be obtained:

\[
k_a [\text{W/m.K}] = 0.155 \times 10^{-3} (T_{\text{film}}[\text{K}])^{0.90} \quad 250 \text{ K} \leq T_{\text{film}} \leq 350 \text{ K} \quad (4-24)
\]

**Justification**

<table>
<thead>
<tr>
<th>( T_{\text{film}} ) [K]</th>
<th>( k_a ) [10^{-3} W/m.K]</th>
<th>( Pr_a ) [-]</th>
<th>( Pr_c ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>18.2 (18.1)</td>
<td>0.737</td>
<td>0.748</td>
</tr>
<tr>
<td>250</td>
<td>22.3 (22.3)</td>
<td>0.720</td>
<td>0.731</td>
</tr>
<tr>
<td>300</td>
<td>26.3 (26.3)</td>
<td>0.707</td>
<td>0.718</td>
</tr>
<tr>
<td>350</td>
<td>30.2 (30.0)</td>
<td>0.700</td>
<td>0.707</td>
</tr>
</tbody>
</table>

1) Calculated with eq. (4-24); values within parenthesis are copied from [08] table A-4.
2) Copied from [08], table A-4.
3) Determined with eq. (4-27).

**Specific heat capacity of air at constant pressure, \( c_{p,a} \)**

It appears that the specific heat capacity of air within the temperature range of 250 - 350 K is practically - to within a margin of less than $\frac{1}{2}$ % - independent of temperature:

\[
c_{p,a} = 1007 \text{ J/kg.K} \quad 250 \text{ K} \leq T_{\text{film}} \leq 350 \text{ K} \quad (4-25)
\]

**Number of Prandtl for air, \( Pr_a \)**

The number of Prandtl for air is weakly dependent on temperature; see table above.

\[
Pr_a \approx 0.71^{30.1} [-] \quad 250 \text{ K} \leq T_{\text{film}} \leq 350 \text{ K} \quad (4-26)
\]

**Clarification**

Apparently, the temperature dependency of viscosity balances that of heat conduction. From the eqn. (4-23-c), (4-24) and (4-25) it follows that:

\[
Pr_a = 1.27 / T_{\text{film}}[\text{K}]^{0.1} \quad 200 \text{ K} \leq T_{\text{film}} \leq 350 \text{ K} \quad (4-27)
\]

Compare the two columns at the right side of the table given above.

**Comparison of correlations for heat transfer from dive bottle to air contained therein**

The eqns. (4-15) and (4-16) with \( Pr_a = 0.71 \) turn in:

\[
Nu_{\text{lam}} [08] \approx 0.67 \cdot (1 + 0.705 \cdot Gr^{3/4}) \quad \text{all material constants at } P_a \text{ and } T_{\text{film}} \quad Gr < 1.4 \times 10^9 \quad (4-28)
\]

And the set eqn. (4-20) likewise with \( Pr_a = 0.71 \) reads as:

\[
Nu_{\text{sub lam.}} [04] \approx 1 + 0.459 \cdot Gr^{4/5} \quad \text{all material constants at } P_a \text{ and } T_{\text{film}} \quad 1.4 \times 10^9 < Gr < 1.4 \times 10^4 \quad (4-29-a)
\]

\[
Nu_{\text{lam.}} [04] \approx 0.542 \cdot Gr^{3/4} \quad \text{all material constants at } P_a \text{ and } T_{\text{film}} \quad 1.4 \times 10^9 < Gr < 1.4 \times 10^9 \quad (4-29-b)
\]

\[
Nu_{\text{turb.}} [04] \approx 0.116 \cdot Gr^{5/6} \quad \text{all material constants at } P_a \text{ and } T_{\text{film}} \quad 1.4 \times 10^9 < Gr < 1.4 \times 10^{12} \quad (4-29-c)
\]

In the opposite fig. 06 the number of Nusselt viz. the eqn. (4-28) and (4-29-a through c) are plotted against that of Grashof. The differences are small in particular for a large part of the interval of Grashof number: $10^3 - 10^9$.

**Note**

\( Nu \) number shows a discontinuity at the transition of a laminar to a turbulent flow, at \( Gr = 1.4 \times 10^9 \).

**Fig. 06** Heat transfer correlations for air in case of natural flow along a vertical plate.
At this transition value of Gr, Nu\textsubscript{ turb. cor.} \approx 0.094 \cdot Gr^{\frac{2}{5}} all material constants at P\textsubscript{a} and T\textsubscript{ film} \quad 1.4 \, 10^9 \leq Gr \leq 1.4 \, 10^{12} \quad (4-29-d)

Combining the eqn. (4-04) and the set (4-29) the following expression for the heat transfer coefficient \( h_{b,a} \) is obtained for further elaborations:

\[
 h_{b,a} = (k_b/L) \cdot \text{Nu} = (k_b/L) \cdot (c_1 + c_2 \cdot \text{Gr}^3)
\]

(4-30-a)

in which:
- regime I: \quad 1.4 \, 10^4 \leq Gr \leq 1.4 \, 10^6: \quad c_1 = 1, \quad c_2 = 0.459 \text{ en } \Gamma = \frac{1}{4} \quad (4-30-b)
- regime II: \quad 1.4 \, 10^4 \leq Gr \leq 1.4 \, 10^6: \quad c_1 = 0, \quad c_2 = 0.542 \text{ en } \Gamma = \frac{1}{4} \quad (4-30-c)
- regime III: \quad 1.4 \, 10^9 \leq Gr \leq 1.4 \, 10^{12}: \quad c_1 = 0, \quad c_2 = 0.116 \text{ en } \Gamma = \frac{3}{10} \quad (4-30-c)

Regime I proves to be of no practical meaning. With the eqn. (4-24), (4-30-a) and (4-30-b), L \equiv H\textsubscript{ eff.} = 0.549 m and T\textsubscript{ film} \approx 300 K, it is found that: \( h_{b,a, \text{ sub lam.}} = 0.048 \, (1 + 0.459 \, \text{Gr}^{\frac{2}{5}}) \)

In other words, the maximum value of the heat transfer coefficient in the concerned Gr-interval occurs at Gr = 1.4 \, 10^4, and is equal to: \( h_{b,a, \text{ sub lam. max.}} \approx 0.29 \, \text{W/m}^2 \cdot \text{K} \). This value is that small that the corresponding heat transfer effects can be neglected all together.

Making use of the numerical thermal physical data of air (see the eqn. (4-18), (4-19), (4-21), (4-22), (4-23-c) with \( g = 9.807 \, \text{m/s}^2 \) and \( L = 0.549 \, \text{m} \) the number of Grashof (see eq. 4-07) equals:

\[
\text{Gr} = 3.26 \cdot 10^8 \frac{P_a^2 \cdot |\Delta T| \cdot \cos \alpha}{(T_{\text{ film}}/100)^{5,6}} \cdot \frac{P_a}{\text{bar}} \text{ and } T_{\text{ film}}[K]
\]

(4-31)

As \( h_{b,a} \) is inversely proportional to L (= H\textsubscript{ eff.}) - see eq. (4-30-a) - and Gr proportional to L\textsuperscript{3} - see eq. (4-07) - the dependency on L for the flow regime II and III - in both cases c\textsubscript{1} = 0 - is small resp. nil.

**Engineering guess of heat transfer coefficient between dive bottle and contained air**

The heat transfer coefficient as given in the preceding sub-heading under eq. (4-30-a) is valid for an inclined vertical plate. It is obvious that the convection flow within the dive bottle does not resemble that along an inclined plate. Hereinafter two intuitive adaptations will be elaborated in order to end up with a more conceivable approximation of the heat transfer process.

In case \( d_{\text{int.}} \to \infty \) the resemblance of the dive bottle with a flat plate will wash. However, for \( d_{\text{int.}} \to 0 \) heat transfer between bottle and air therein will vanish; \( h_{b,a} \to 0 \). Apparently, the driving parameter that plays a decisive part in the heat transfer process is: \( L / d_{\text{int.}} \). This quantity must be dimensionless, indeed. Intuitively, it is thus reasonable to replace L\textsuperscript{3} in Gr-number by:

\[
L^3 / (1+L/d_{\text{int.}})
\]

or in other words (vert. refers to vertical):

\[
\text{Gr}_{\text{vert.}} = \text{Gr}_{\text{eq. (4-07) subsequently (4-31) / (1+L/d_{\text{int.}})}}
\]

(4-32-a)

The second adaptation concerns the case that the dive bottle is not in an upright but in a horizontal position, viz. \( \alpha = 90^0 \). Instead of L\textsuperscript{3}, half the circumference of the dive bottle raised to the 3\textsuperscript{th} power, \( (\frac{1}{2} \pi \cdot d_{\text{int.}})^3 \) comes into view. This being too optimistic, is replaced by d\textsuperscript{3}_{int.}, which seems more reasonable. Therefore:

\[
\text{Gr}_{\text{hor.}} = \text{Gr}_{\text{eq. (4-07) subsequently (4-31) \cdot (d_{\text{int.}}/L)^3}}
\]

(4-32-b)

If it is finally assumed, that Gr\textsubscript{a} between the values of Gr\textsubscript{vert.} and Gr\textsubscript{hor.} is shaped elliptically - a linear course is of course excluded as this will results in Gr\subscript{0 < \alpha < 90} values lower than the lowest value of Gr\textsubscript{vert.} and Gr\textsubscript{hor.} - than the following result is obtained (leaving out the analytical manipulations):

\[
\text{Gr}_{\alpha} = \frac{1}{\sqrt{\left(\frac{\sin \alpha}{\text{Gr}_{\text{hor.}}}\right)^2 + \left(\frac{\cos \alpha}{\text{Gr}_{\text{vert.}}}\right)^2}}^{1/2}
\]

(4-33)
With \( L = H_{\text{eff}} = 0.549 \text{ m}, \ d_{\text{int.}} = 0.1742 \text{ m} \) and the eqn. (4-32-a) and (4-32-b) in stead of eq. (4-31) the following expression is obtained for Grashof number:

\[
\text{Gr}_\alpha = 3.26 \cdot 10^9 \frac{P_a^2 \cdot |\Delta T|}{(T_{\text{film}}/100)^{4.6}} \left( \left(1 + L/d_{\text{int.}} \right) \cos \alpha \right)^2 + \left(\frac{L}{d_{\text{int.}}} \right)^3 \sin \alpha \right)^2 \right)^{-1/2}
\]

Therefore:

\[
\text{Gr}_{\text{hor.}} = 0.104 \cdot 10^9 \frac{P_a^2 \cdot |\Delta T|}{(T_{\text{film}}/100)^{4.6}} \tag{4-34-a}
\]

\[
\text{Gr}_{30} = 0.203 \cdot 10^9 \frac{P_a^2 \cdot |\Delta T|}{(T_{\text{film}}/100)^{4.6}} \tag{4-34-b}
\]

\[
\text{Gr}_{\text{vert.}} = 0.786 \cdot 10^9 \frac{P_a^2 \cdot |\Delta T|}{(T_{\text{film}}/100)^{4.6}} \tag{4-34-c}
\]

Making use of the set of eq. (4-30) the following result is obtained for the heat transfer coefficient between dive bottle and air contained therein:

\[
\text{II} \quad h_{b-a, \text{lam.} \alpha} \ [W/m^2 \cdot K] \approx 9.66 \cdot 10^{-3} \cdot \text{Gr}_\alpha^{1/4} \cdot \left( \frac{T_{\text{film}} [K]}{100} \right)^{0.9} \quad \text{Gr}_\alpha < 1.4 \cdot 10^9 \tag{4-35-a}
\]

\[
\text{III} \quad h_{b-a, \text{turb.} \alpha} \ [W/m^2 \cdot K] \approx 2.07 \cdot 10^{-3} \cdot \text{Gr}_\alpha^{1/3} \cdot \left( \frac{T_{\text{film}} [K]}{100} \right)^{0.9} \quad \text{Gr}_\alpha > 1.4 \cdot 10^9 \tag{4-35-b}
\]

**Numerical examples**

i) Given \( T_{\text{film}} = 265 \text{ K}, P_a = 100 \text{ bar} \) and \( \Delta T = 25 \text{ °C} \)

<table>
<thead>
<tr>
<th>Inclination</th>
<th>( \text{Gr}_\alpha [10^{12}] )</th>
<th>( h_{b-a, \text{turb.}} [W/m^2.K] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizontal</td>
<td>0.294</td>
<td>33.1</td>
</tr>
<tr>
<td>30\text{°} inclined</td>
<td>0.574</td>
<td>41.4</td>
</tr>
<tr>
<td>vertical</td>
<td>2.22*</td>
<td>64.9</td>
</tr>
</tbody>
</table>

* In fact outside Gr-interval

ii) Given \( T_{\text{film}} = 275 \text{ K}, P_a = 200 \text{ bar} \) and \( \Delta T = 1/2 \text{ °C} \)

<table>
<thead>
<tr>
<th>Inclination</th>
<th>( \text{Gr}_\alpha [10^{12}] )</th>
<th>( h_{b-a, \text{turb.}} [W/m^2.K] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizontal</td>
<td>0.040</td>
<td>17.6</td>
</tr>
<tr>
<td>30\text{°} inclined</td>
<td>0.077</td>
<td>21.9</td>
</tr>
<tr>
<td>vertical</td>
<td>0.300</td>
<td>34.4</td>
</tr>
</tbody>
</table>

iii) It is noted that the estimated values of \( h_{b-a} \) are certainly not on the too low side, on the contrary; see in this connection table 05.

4.6 Thermal decay 'constant' and half-value time of dive air

The thermal decay constant of dive air \( a_a \) see eq. (3-17) is anything but constant as both the air mass, \( m_a \) contained in the dive bottle as well as the free convection heat transfer coefficient, \( h_{b-a} \) are changing continuously. Recalling eq. (3-17):
\( a_a[1/s] = (h_{b-a} \cdot F_a) / (m_a \cdot c_a) \)

in which:
- \( m_a[kg] = \rho_a \cdot V_a = 4.18 \cdot \frac{P_a[bar]}{T_a[K]} \)  
  (4-36-a)
- \( c_a = c_v, a = c_{p, a} / \gamma_a = 1007 / 1.40 = 719 [J/kg.K] \)  
  (4-36-b)
- \( F_a = 0.324 \, m^2 \)  
  (4-36-c)

Substitution results in:

\[ a_a[1/s] = 0.108 \cdot 10^3 \frac{P_a[bar]}{T_a[K]} \cdot h_{b-a}[W/m^2K] \]  
(4-37)

in which \( T_a \) can be expressed in the driving quantities \( T_{film} \) and \( |\Delta T| \):

\[ T_a = T_{film} \left(1 - \frac{|\Delta T|}{2 \cdot T_{film}[K]} \right) \]  
(4-37-a)

**Numerical example**

**Given**

Order of magnitude of \( h_{b-a} = 50 \, W/m^2.K \) see afore given numerical examples.

**Question**

Half-time value of dive air?

**Answer**

Assumed \( h_{b-a} = 50 \, W/m^2.K \) corresponds with: \( P_a \approx 100 \, bar \) en \( T_a = 265 \, K \).

From the eqn. (3-24-a) and (4-37) the following result is obtained: \( \tau_{a[s]} = \ln(2/a_s) \approx 48 \, s \).

The half value time of dive air follows from the eqs. (3-24-a) and (4-37):

\[ \tau_{a,i}[s] = 6.418 \cdot 10^3 \frac{P_a[i,+] [bar]}{T_{A,i,+}[K] \cdot h_{b-a,i}[W/m^2K]} \]  
(4-38)

**Note**

Expressing \( \tau_a \) in the driving quantities yields the following results; combining the eq. (4-34), (4-35-a) and (4-38) and likewise the eq. (4-34), (4-35-b) and (4-38):

\[ \begin{align*}
\text{II} \quad & \tau_{a, \text{lamb}}[s] = \frac{27.8}{1 - \frac{\Delta T}{2 \cdot T_{film}[K]}} \left( \frac{P_a[bar]}{(T_{film}[K]/100)^{1.5}} \right)^{1/2} \left( \frac{(4.15 \cdot \cos \alpha)^2 + (31.3 \cdot \sin \alpha)^2}{|\Delta T|^{1/4}} \right)^{1/8} \\
& \approx 27.8 \left( \frac{P_a[bar]}{(T_{film}[K]/100)^{1.5}} \right)^{1/2} \left( \frac{(4.15 \cdot \cos \alpha)^2 + (31.3 \cdot \sin \alpha)^2}{|\Delta T|^{1/4}} \right)^{1/8} \tag{4-39-a}
\end{align*} \]

\[ \begin{align*}
\text{III} \quad & \tau_{a, \text{turb}}[s] = \frac{20.9}{1 - \frac{\Delta T}{2 \cdot T_{film}[K]}} \left( \frac{P_a[bar]}{(T_{film}[K]/100)^{1.1}} \right)^{1/3} \left( \frac{(4.15 \cdot \cos \alpha)^2 + (31.3 \cdot \sin \alpha)^2}{|\Delta T|^{1/3}} \right)^{1/6} \\
& \approx 20.9 \left( \frac{P_a[bar]}{(T_{film}[K]/100)^{1.1}} \right)^{1/3} \left( \frac{(4.15 \cdot \cos \alpha)^2 + (31.3 \cdot \sin \alpha)^2}{|\Delta T|^{1/3}} \right)^{1/6} \tag{4-40-a}
\end{align*} \]

**Numerical examples**

**Given**

\( T_{film} = 265 \, K, \, P_a = 100 \, bar \) and \( \Delta T = 25 \, ^\circ C \)

**Question**

\( \tau_{b,a} \) for \( \alpha = 90^\circ, 30^\circ \) and \( 0^\circ \)

**Answer**

For \( h_{b-a, \text{turb}} \) see one before last example and then proceed with eqn. (4-38), (4-37) and (3-24-a). Of course eq. (4-40-b) can be used as well.
Inclination | $h_{b,a \text{- turb.}}$ [W/m².K] | $T_a$ [K] | $a_a$ [1/s] | $\tau_a$ [s]  
--- | --- | --- | --- | ---  
horizontal | 33.1 | 253 | 9.03 $10^{-3}$ | 77  
30° inclined | 41.4 | 253 | 11.3 $10^{-3}$ | 61  
vertical | 64.9 | 253 | 17.7 $10^{-3}$ | 39  

Given $T_{\text{film}} = 275$ K, $P_a = 200$ bar and $\Delta T = \frac{1}{2}$ °C

**Answer**

| Inclination | $h_{b,a \text{- turb.}}$ [W/m².K] | $T_a$ [K] | $a_a$ [1/s] | $\tau_a$ [s]  
--- | --- | --- | --- | ---  
horizontal | 17.6 | 275 | 2.61 $10^3$ | 266  
30° inclined | 21.9 | 275 | 3.25 $10^3$ | 213  
vertical | 34.4 | 275 | 5.11 $10^3$ | 136  

In conclusion, the estimated half-value time characterizing the heat transfer process between dive bottle and the air contained therein lead up to impressive lengthy analytical expressions. However, the author hardly dares to assign an inaccuracy margin to the estimated values of $\tau_a$. Margins in the order of ± 50% will not be surprising. Only by performing proper experiments, more reliable results can be guaranteed.

### 4.7 Ratio of heat capacity of dive air versus that of dive bottle

The ratio between the heat capacity of dive air and that of dive bottle, $\lambda$ follows from the eqn. (3-19) and (4-36-a en -b) with $m_b = 16.5$ kg and $c_b = 880$ J/kg.K:

$$\lambda^{(+)} = \frac{(m_a \cdot c_a)^{(+)} / (m_b \cdot c_b)}{0.207 \cdot P_{a, i^{(+)}} \text{[bar]} / T_{a, i^{(+)}} \text{[K]}} \quad (4-41)$$

**Numerical example**

- **Given** $T_{a, i^{(+)}} = 273.15$ K and $P_{a, i^{(+)}} = 180$ bar
- **Question** Determine $\lambda$
- **Answer** eq. (4-41) $\Rightarrow \lambda = 0.136$
- **Comment** $\lambda$-range: 0 - 0.15

$\lambda$-range: 0 - 0.15

Expressing $\lambda$ in the driving quantities; combination of the eqn. (4-41) and (3-38) yields:

$$\lambda^{(+)} [-] = 0.207 \frac{P_{a, i^{(-)}}}{T_{a, i^{(-)}}} \left( \frac{T_{a, i^{(+)}}}{T_{a, i^{(-)}}} \right)^{1/(\gamma - 1)} = 0.207 \frac{P_{a, i^{(-)}} \text{[bar]}}{T_{a, i^{(-)}} \text{[K]}} \left( 1 - \frac{\Delta T_{a, i}}{T_{a, i^{(-)}} \text{[K]}} \right)^{2.5} \quad (4-42)$$
5 RESULTS
5.1 Introduction
In view of the large amount of parameters and the large degree of uncertainties in the heat transfer coefficients three dive scenarios will be assessed. These are:
I A pair of stylized dives each of which for reasons of simplicity, with only one inspiration. This scenario is infused in order to get some insight/feeling in the pressure and temperature progress of the air within a dive bottle on the one hand and the temperature progress of the dive bottle on the other hand;
II Simplified dive scenarios;
III Realistic dive scenarios.

In order to avoid any misunderstandings all data, initial conditions and parameters will be summarized before the calculation results are presented.

5.2 Two stylized dives each of which with only one act of inspiration; scenario I

Scenario I
It concerns a pair of stylized dives to a depth of 20 m. During one of these dives, inhalation takes place at the beginning of the dive while in the other case the one inhalation takes place halfway during diving. The dives are referred to as I-a and I-b respectively. In both cases the beginning of the dive takes place at the moment in time of t = 0.

Fixed data, initial conditions, parameters and assumptions; scenario I-a and I-b

Fixed data
\[ v_{\text{des}} = 20 \text{ m/min} \] (input data)
\[ D_{\text{bottom}} = 20 \text{ m} \] (input data)
\[ \alpha = 30^0 \] (assumed dive inclination)
\[ v_d = 23.09 \text{ m/min} \] (see eq. (2-10))
\[ T_w = 26^0 \text{ C} \] (input data)
\[ T_{\text{ref.}} = 273.15 \text{ K } (0 ^0 \text{ C}) \] (by definition)
\[ P_{\text{ref.}} = 1 \text{ bar} \] (by definition)
\[ i : 0, 1, ... n \] "breathing index"

Material and construction data of dive bottle and thermal physical data of water and air is compiled in annex B.

Initial conditions
\[ P_{a,0} = 200 \text{ bar} \] (input data)
\[ T_{b,0} = 32^0 \text{ C} \] (input data)
\[ T_{a,0} = 32^0 \text{ C} \] (input data)

Parameters
i. At the moment in time t = 0 for scenario I-a alternatively at t = 30 s for scenario I-b only one inhalation of 100 nℓ takes place, notwithstanding that this is from the physiological point of view impossible. In order to get some idea of the effect of the magnitude of such an inspiration, calculations are performed with:
\[ \delta V_0 = 75, 100 \text{ and } 125 \text{ nℓ} \] for scenario I-a and \[ \delta V_{30} = 75, 100 \text{ and } 125 \text{ nℓ} \] for scenario I-b.

Clarification
The chosen value of 100 nℓ is prompted by the numerical examples given in paragraph 3.1; \[ \delta V = 13 + 93 \approx 100 \text{ nℓ}. \] In view of the uncertainty in the required air consumption RMV, \[ \delta V \] is treated as a parameter, viz. the expected uncertainty of the air consumption for a certain required amount of work/swimming output is discounted for by choosing \[ \delta V \] as a parameter.

ii. At a descend speed of 20 m/min at an inclination of \[ \alpha = 30^0 \], the \[ h_{w-b} \] uncertainty range is estimated to the best of the author's knowledge in paragraph 4.4 under the heading 'Adopted parameter values of \[ h_{w-b} \]'. Therefore scenario I-a is chosen with \[ h_{w-b} \] as parameter. Uncertainties in the value of \[ h_{b-a} \] on the other hand are dealt with in scenario I-b. Thus:
- scenario I-a: \( h_{w-b} = 480, 700 \) of 930 W/m²K;
\( \tau_a \) : nominal calculated value by means of the eqs. (3-17) and (4-37);
- scenario I-b: \( h_{w-b} = 700 \) W/m²K;
\( \tau_a \) : nominal calculated value by means of the abovementioned equations for \( \tau_a \) and a value of 0.7 resp. 1.3 times that nominal value.

**Assumptions**

For reasons of simplification, it is assumed that the heat transfer coefficients, \( h_{b-a} \) between bottle and dive air contained therein is constant during the considered time span, or in other words for:
- scenario I-a: constant and equal to the value at the point in time of \( t = 0 \);
- scenario I-b: constant and equal to the value at the point in time of \( t = 0 \) for \( 0 < t < 30 \) s;
   constant and equal to the value at the point in time of \( t = 30 \) s for \( t \geq 30 \) s.

**Compilation of equations used in case of scenario I-a**

Parameters are denoted in Italian symbols.

\[
\Delta T_a,0 = 305.15 \cdot \{1 - (1 - 0.466 \cdot 10^{3} \cdot \delta V_0)\}^{0.4} \approx 0.0568 \delta V_0 \quad \text{[K] or [°C]} \quad \text{(see eq. (3-37))}
\]
\[
T_a,0(+) = 305.15 - \Delta T_a,0 \quad \text{[K]} \quad \text{(see eq. (3-28))}
\]
\[
P_a,0(+) = 200 + (1 - \Delta T_{a,0} / 305.15)^{3.5} \quad \text{[bar]} \quad \text{(see eq. (3-38))}
\]
\[
\tau_b = 24.1 \cdot 10^{3} / h_{w-b}[\text{W/m}^2\text{K}] \quad \text{[s]} \quad \text{(see eq. (4-03))}
\]
\[
\Delta T_0 = \Delta T_{a,0} \approx 0.0568 \delta V_0 \quad \text{[K or °C]} \quad \text{(see eq. (4-18))}
\]
\[
T_{\text{film}} = 305.15 - 0.0284 \delta V_0 \quad \text{[K]} \quad \text{(see eq. (4-17))}
\]
\[
\text{Gr}_{30} = 0.239 \cdot 10^{9} \frac{P_{A,0(+)}}{(T_{\text{film}}/100)^{4.6}} 19 \quad \text{[-]} \quad \text{(see eq. (4-34-b))}
\]
\[
\begin{align*}
\text{h}_{b-a,\text{turb.}a}[\text{W/m}^2\cdot\text{K}] & \approx 2.07 \cdot 10^{-3} \cdot \text{Gr}_{30}^{1/3} \cdot \left(\frac{T_{\text{film}}}{100}\right)^{0.9} \quad \text{[Gr}_{a}>1.4 \cdot 10^{-9}] \quad \text{(see eq. (4-35-b))}\n\end{align*}
\]
\[
\tau_a,0(+) = 4.21 \cdot 10^{3} \cdot (1 - \Delta T_{a,0} / 305.15)^{2.5} / h_{b-a}[\text{W/m}^2\text{K}] \quad \text{[s]} \quad \text{(see eqs. (3-17) and (4-38))}
\]
\[
\lambda_{\text{el}}(+) = 0.136 \cdot (1 - \Delta T_{a,0} / 305.15)^{2.5} \quad \text{[-]} \quad \text{(see eq. (4-42))}
\]
\[
C_{1.1} = (6 - \Delta T_{a,0}) / F_2 \cdot F_3 \quad \text{[°C]} \quad \text{(see eq. (3-22))}
\]
\[
C_{2.1} = 6 - \Delta T_{a,0} / C_{1.1} \quad \text{[°C]} \quad \text{(see eq. (3-23))}
\]
\[
\Delta P_{a}(t) = P_{a,0(+) - P_a(t)} = 200 - (T_{a}(t)/T_{a,0(+)}) \cdot P_{a,0(+) \text{ bar} t \geq t_{0(+)}} \quad \text{(see eq. (3-40))}
\]

The course of temperature of the dive bottle and that of the dive air therein follows from the eqs. (3-21) resp. (3-20) in which, for these scenarios \( T_{\text{resp}} \) must be replaced by \( t \); see the eqs. (3-41) and (3-42) also.

**Compilation of numerical data; scenario I-a**

See table 07.

**Results stylized dive I-a**

The results are shown in the figs. 07 up to 09 inclusive. By mistake one could think that \( T_a \) (the temperature of dive air in the dive bottle) can not become greater than \( T_b \). However, this can be the case indeed. In the opposite fig. 10 it is shown how the temperature progresses over a long period of time for \( \delta V = 75 \) nl and \( h_{w-f} = 700 \) W/m²K. At the moment in

---

Fig. 10 \( T_b \) and \( T_a \) in function of time in case of scenario I-a with \( \delta V_0 = 75 \) nl and \( h_{w-b} = 700 \) W/m²K.
time on which \( T_a \) equals \( T_b \), the air temperature will not (cannot) increase anymore, but reaches a maximum value. For the present scenario I-a with \( \delta V_0 = 75 \text{nL} \) and \( h_{w-b} = 480, 700 \) and \( 930 \text{ W/m}^2\text{K} \) the maximum dive air temperature (being very close to 28.2 °C) occur at 67, 50 resp. 39 s approximately (see fig 07). Furthermore, both bottle and air temperatures approach the seawater temperature of 26 °C asymptotically. In this connection it should be recalled that this scenario is only hypothetical as the act of inhalation occurs once-only and that, at the same time, it is assumed for reasons of simplicity that the heat transfer coefficient from dive bottle to the air contained therein remains constant throughout the course of time. This is for sure not the case.

Going back to the concerned figures (07, 08 and 09) it is seen that the trends in the progress of temperature and pressure comply with one's expectations. The larger the volume of the once-only inhalation, the larger the drop in air temperature and dive bottle pressure; see table 07: second row for \( \Delta T_{a,0}(4,3, 5.7 \text{ and } 7.1 \text{ °C}) \) and fifth row for \( \Delta P_{a,0}(9.6, 12.7 \text{ and } 15.8 \text{ bar}) \).

In addition, from the figures it appears, again as expected that if the heat transfer coefficient between water and dive bottle, \( h_{w-b} \) increases from 480 → 700 → 930 \text{ W/m}^2\text{K} \) the corresponding temperature changes of the dive bottle become larger. This is primarily reflected also, in the corresponding \( \tau_{0} \)-values\(^2\) of 50, 34 resp. 26 s (see table 07, 11th row). From fig. 08 it can be ascertained that the temperature decrease in \( T_b \) yields 'true' half-value times of 47.6 and 25.3 s, for \( h_{w-b} \) values of 480 resp. 930 \text{ W/m}^2\text{s}, implying (not surprisingly after all) that the ratio of the half-value times 1.88 (= 47.6/25.3) is strongly correlated to the inversed ratio of the heat transfer coefficients 1.93 (= 930/480). From the physical point of view, this means that the contained air in the dive bottle hardly has any influence on the temperature progress of the dive bottle.

The parametric influence of the heat transfer coefficients \( h_{w-b} \) on the course of the dive air temperature corresponds with the driving quantity \( |\Delta T| = |T_b - T_a| \) in the number of Grashof. The larger \( |\Delta T| \) at a certain moment in time, the faster \( T_a(t) \) changes in function of time, viz. thus in catching up with \( T_b(t) \); see left parts of the set of figs 07, 08 and 09.

Finally, it is seen that the progress of the pressure decrease corresponds with that of the progress of dive air temperature, brought about by isochoric effect: increasing dive air temperature corresponds with decreasing pressure-decrease. This explains the parameter effect of the \( h_{w-b} \) on the decrease in pressure-decrease of the dive air.

**Compilation of equations used in case of scenario I-b\(^2\)**

**Time interval: 0 ≤ [s] ≤ 30 (c)**

\[
\begin{align*}
\delta V_0 &= 0 \rightarrow \Delta T_{a,0} \approx 0.0568 \delta V_0 = 0 \quad [\text{K or } 0\text{°C}] \\
T_{a,0(+)} &= 305.15 - \Delta T_{a,0} = 305.15 \\
P_{a,0(+)} &= 200 \times (1 - \Delta T_{a,0} / 305.15)^{3.5} = 200 \quad [\text{bar}] \\
\tau_b &= 24.1 \times 10^3 / h_{w-b}[\text{W/m}^2\text{K}] \\
\Delta T_0 &= \Delta T_{a,0} = 0 \quad [\text{K or } 0\text{°C}] \\
T_{film} &= 305.15 - 0.0284 \delta V_0 = 305.15 \quad [\text{K}] \\
Gr_30 &= 0.203 \times 10^9 \frac{P_{a,0(+)}^2}{(T_{film}/100)^{4.6}} \frac{|\Delta T_0|}{\tau_a} = 0 \quad [-] \\
\lambda_{b-a} &= (Gr_30)^{1/5} \times 1/ \tau_b = 0 \quad [\text{W/m}^2\text{K}] \\
\tau_a& = 4.21 \times 10^3 \times (1 - \Delta T_{a,0} / 305.15)^{2.5} / \lambda_{b-a} = \infty \quad [\text{s}] \\
\lambda_{a} &= 0.136 \times (1 - \Delta T_{a,0} / 305.15)^{2.5} = 0.136 \quad [-] \\
C_{1,1} &= (6 - \Delta T_{a,0}) \times F_2 - 6 / F_3 = 6 (F_2 - 1) / F_3 \quad [\text{°C}] \\
C_{2,1} &= 6 - \Delta T_{a,0} - C_{1,1} = 6 - C_{1,1} \quad [\text{°C}]
\end{align*}
\]

After some tedious mathematical manipulation, brought about by the limit transition of \( \tau_{a,0(+)} \rightarrow \infty \) and the indefiniteness of \( \infty / \infty \) and \( 0 / \infty \) the following predictable results turn up:

\(^2\) By definition of eq. (3-24-b) the half-value time of the dive bottle.

\(^2\) See scenario I-a for the concerned equations.
\[ T_a = 32 \text{ and } T_b = 26 + 6 \cdot 2^{4/3} \text{[°C]} \]

**Time interval: 30(+) ≤ [s]**

\[
\begin{align*}
\Delta T_{a,30} & = 305.15 \cdot \{ 1 - (1 - 0.466 \cdot 10^{-3} \delta V_{30})^{0.4} \} \approx 0.0568 \delta V_{30} \text{[K]} \\
T_{a,30(+)} & = 305.15 - \Delta T_{a,30} \text{[K]} \\
P_{a,30(+)} & = 200 \cdot (1 - \Delta T_{a,30} / 305.15)^{3.5} \text{[bar]} \\
\tau_b & = 24.1 \cdot 10^3 / h_{w-b}[\text{W/m}^2\text{K}] \text{[s]} \\
\Delta T_{30} & = T_{b,30} - T_{a,30(+)} = T_{b,30}[\text{C}] - 32 + 0.0568 \delta V_{30} \text{[°C]} \\
T_{\text{film},30} & = (T_{b,30}[\text{K}] + 305.15 - 0.0568 \delta V_{0}) / 2 \text{[K]} \\
Gr_{30} & = 0.203.10^{6} \frac{P_{a,30(+)}^2 |\Delta T_{30}|}{(T_{\text{film},30}/100)^{4.6}} \\

h_{b-a,\text{turb.}} & \approx 2.07 \cdot 10^{-3} \left( \frac{\text{Gr}_{30}^{1/3} \cdot T_{\text{film}}[\text{K}]}{100} \right)^{0.9} \text{[W/m}^2\text{K]} \\
\tau_{a,30(+)} & = 4.21 \cdot 10^3 \cdot (1 - \Delta T_{a,30} / 305.15)^{2.5} / h_{b-a}[\text{W/m}^2\text{K}] \text{[s]} \\
\lambda_{30(+)} & = 0.136 \cdot (1 - \Delta T_{a,30} / 305.15)^{2.5} \text{[-]} \\
C_{1,31[\text{°C}]} & = \{ (6 - \Delta T_{a,30}) \cdot F_2 - (T_{b,30} - 26) \} / F_3 \text{[°C]} \\
C_{2,31[\text{°C}]} & = 6 - \Delta T_{a,30} - C_{1,31} \]

**Compilation of numerical data; scenario I-b**

See table 08.

**Results stylized dive I-b**

The results are shown in fig. 11. Noticeable is that, the influence of the magnitude of the heat transfer capacity/coefficient from dive bottle to the air therein and vice versa, on the temperature course of the dive bottle T_b is nil; the curves with \( h_{b-a} \) as a parameter, calculated with eq. (3-21) nearly coincide with each other. This is once more illustrated in the opposite fig. 12, showing results with 'extreme' values of \( h_{b-a} \); the nominal calculated value of 28.5 W/m²K, a value ten times lower and ten times higher: viz. 2.85 resp. 258.5 W/m²K. Recall that all calculations are performed with \( \delta V_t = 100 \text{ nl} \) at the moment in time of \( t = 30 \text{ s} \) and \( h_{w-b} = 700 \text{ W/m²K} \). The differences in \( T_b(t) \) are that small, that calculation and/or plotting inaccuracies might come into play. However, this is not the case as is seen in the lower part of fig. 12 where the difference in the dive bottle temperatures, \( \Delta T_b \) shows a conceivable smooth progress. \( \Delta T_b \) is defined as:

\[
\begin{align*}
\Delta T_{b;2.85} & \equiv T_b, \text{ with } h_{b-a} = 2.85 - T_b, \text{ with } h_{b-a} = 28.5 \\
\Delta T_{b;285} & \equiv T_b, \text{ with } h_{b-a} = 285 - T_b, \text{ with } h_{b-a} = 28.5
\end{align*}
\]

As seen in the upper part of fig. 12 the adopted heat transfer coefficients, \( h_{b-a} \) influences the progress of \( T_a \) nearly as limiting values, as if \( h_{b-a} \rightarrow 0 \text{ resp. } \rightarrow \infty \).

**Fig. 12** \( T_b, T_a \) and \( \Delta T_b \) in function of time in case of scenario I-b with \( \delta V_{30} = 100 \text{ nl} \), \( h_{w-b} = 700 \text{ W/m²K} \) and \( h_{b-a} \) as parameter (with extreme values).
If the heat transfer from bottle to dive air is small (see \( T_b(100;2,85) \)-curve), the air temperature after the adiabatic temperature drop, hardly changes while on the other hand the air temperature changes rapidly to catch up with the dive bottle temperature for large values of the heat transfer from bottle to dive air (see \( T_a(100;285) \)-curve).

Though hardly seen in the upper part of fig. 12 but clearly in the lower part is that the drop-off in bottle temperature slightly increases the larger the heat transfer from bottle to contained air.

\textit{Note}

Going back to the set of figs. 11 it is, though not clearly seen that the \( T_c \)-curves intersecting each other after crossing the corresponding \( T_b \)-curves. This is seen clearly, in the blown up part of fig. 11 for \( \delta V_{30} = 100 \) nl; see opposite fig. 13.

This is conceivable indeed, as the larger, the heat transfer capacity/coefficient from air to bottle, ultimately the faster the bottle temperature will be approached.

5.3 Simplified scenario

\textbf{Scenario II}

The scenario at hand deals with dives to 20 m, also as in case of scenario I. This time however, the dives are more realistic, though still schematized so as seen in fig.01. A dive starts at the moment in time of \( t = 0 \) s. The corresponding RMV is determined on basis of data compiled in paragraph 2.2, in which RMV is expressed in function of the diver's swimming velocity.

In view of the large number of degrees of freedom this scenario is limited to the so called base line case with only one value for the parameter \( h_{w-b} \) (= 700 W/m\(^2\)K) and RMV (= 27,87 ℓ during descend). For the influence of other values of these parameters, reference is made to scenario III.

In the present scenario, the initial air pressure at the onset of a dive is dealt with as a parameter, in order to obtain some feeling of the influence of this pressure.

\textbf{Dive profile}

Though the present scenario remains a simplification, it is assumed that 12 s before the commencement of descending the heat transfer process starts. The diver swims at surface with a speed of say \( v_{d, \text{surface}} \) of 10 m/min with air breathing trough the snorkel.

In order to get some feeling what happens after descending it is further assumed that the diver swims horizontal at bottom depth with half the descending velocity, thus: \( v_{d, \text{bottom}} = 12 \) m/min.

The first inspiration with air from the dive bottle takes place at the moment in time of \( t = 0 \):

\[ \delta V_{a,0} = 2.75 \text{ ℓ}. \]

\textbf{Fixed data, initial conditions, parameters and assumptions; scenario II-a and II-b}

\textit{Fixed data}

\begin{align*}
  v_{d, \text{surface}} & = 10 \text{ m/min} & \text{(input data)} \\
  v_{d, \text{bottom}} & = 12 \text{ m/min} & \text{(input data)} \\
  v_{d_{\text{es}}} & = 20 \text{ m/min} & \text{(input data; as scenario I)} \\
  D_{\text{bottom}} & = 20 \text{ m} & \text{(input data; as scenario I)} \\
  \alpha & = 30^\circ & \text{(assumed dive inclination; as scenario I)} \\
  v_d & = 23.09 \text{ m/min} & \text{(see eq. (2-10); as scenario I)} \\
  T_w & = 26 \text{ °C} & \text{(input data; as scenario I)}
\end{align*}
$T_{\text{ref.}} = 273.15 \, ^\circ\text{K} (0 \, ^\circ\text{C})$ (by definition; as scenario I)

$P_{\text{ref.}} = 1 \, \text{bar}$ (by definition; as scenario I)

\(i = 0, 1, \ldots, n = 15\) "breathing index", discretized time steps

\(\rightarrow n = 60 \cdot \text{D}_{\text{bottom}} / (\text{V}_{\text{des.}} \cdot T_{\text{resp.}}) = 15\)

\(\text{V}_{\text{tot}} = 6\frac{1}{2} \, \ell\) (assumed total lung capacity; see table 01)

$T_{\text{resp.}} = 4 \, \text{s}$ (base line value; see further down under assumptions)

$h_{\text{w-f}} = 700 \, \text{W/m}^2\text{K}$ (assumed nominal value for scenario II)

$\text{RMV}_{\text{des.}} = 27.87 \, \text{n}\ell$ (base line value; see further down under assumptions)

\textbf{Initial conditions}

$T_{b,0} = 32 \, ^\circ\text{C}$ (input data)

$T_{a,0(-)} = 32 \, ^\circ\text{C}$ (input data)

\textbf{Parameters}

Scenario II-a: $P_{a,0(-)} = 200 \, \text{bar}$ (input data)

Scenario II-b: $P_{a,0(-)} = 100 \, \text{bar}$ (input data)

\textbf{Assumptions}

1. In order to generate the requested RMV either $V_{\text{work}}$ or the breathing period, $T_{\text{resp.}}$ can be chosen. Whether $V_{\text{work}}$ or $T_{\text{resp.}}$ is chosen does not matter as the quotient of these quantities is fixed by the required value of RMV (see eq. (2-06)). Referring to paragraph 2.2 under the heading 'Redundant dependent variables', the following constant value for the breathing period is chosen:

\[T_{\text{resp.}} = 4 \, \text{s}, \text{unless stated otherwise.}\]

2. In order not to complicate the calculations unnecessarily it is assumed that after $i = 18$ ($t = 72 \, \text{s}$) the calculation time interval equals $4T_{\text{resp.}}$, implying at the same time that as a consequence the corresponding value of $\delta V_{a}$ is four times that large.

The above given assumption are compiled in table 09, in relation to the corresponding time steps.

\textbf{Table 09} Summary of assumptions for dive scenario II

<table>
<thead>
<tr>
<th>$i$</th>
<th>$t_i$ [s]</th>
<th>$\Delta t_{i,i+1}$ [s]</th>
<th>$v_d, i/i+1$ [m/min]</th>
<th>$\text{RMV}_{i,i+1}$ [n/l/min]</th>
<th>$\delta V_{a,i(+)}$ [n/l]</th>
<th>$h_{\text{w-b},i}$ [W/m$^2$K]</th>
<th>$\tau_f, i$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-12</td>
<td>4</td>
<td>10$^2$</td>
<td>0 (14.16)</td>
<td>nil</td>
<td>427</td>
<td>56.5</td>
</tr>
<tr>
<td>-2</td>
<td>-8</td>
<td>4</td>
<td>10</td>
<td>0 (14.16)</td>
<td>nil</td>
<td>427</td>
<td>56.5</td>
</tr>
<tr>
<td>-1</td>
<td>-4</td>
<td>10</td>
<td>0 (14.16)</td>
<td>nil</td>
<td>427</td>
<td>56.5</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>23.09</td>
<td>n.a.$^5$</td>
<td>2.75$^5$</td>
<td>705</td>
<td>34.2</td>
<td></td>
</tr>
<tr>
<td>1, etc.</td>
<td>4</td>
<td>23.09</td>
<td>27.87$^6$$^8$,07</td>
<td>2.97</td>
<td>705</td>
<td>34.2</td>
<td></td>
</tr>
<tr>
<td>t/m 14</td>
<td>56</td>
<td>23.09</td>
<td>27.87$^6$$^8$,07</td>
<td>6.19</td>
<td>705</td>
<td>34.2</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>60</td>
<td>12</td>
<td>27.87$^6$$^8$,07</td>
<td>6.44</td>
<td>476</td>
<td>50.6</td>
<td></td>
</tr>
<tr>
<td>16, etc.</td>
<td>64</td>
<td>12</td>
<td>15.49$^6$$^6$,07</td>
<td>1.10</td>
<td>476</td>
<td>50.6</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>72</td>
<td>12</td>
<td>15.49$^6$$^6$,07</td>
<td>1.10</td>
<td>476</td>
<td>50.6</td>
<td></td>
</tr>
<tr>
<td>19, etc.</td>
<td>76</td>
<td>12</td>
<td>15.49$^6$$^6$,07</td>
<td>1.10</td>
<td>476</td>
<td>50.6</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>140</td>
<td>12</td>
<td>15.49$^6$$^6$,07</td>
<td>12.39</td>
<td>476</td>
<td>50.6</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>156</td>
<td>12</td>
<td>15.49$^6$$^6$,07</td>
<td>12.39</td>
<td>476</td>
<td>50.6</td>
<td></td>
</tr>
<tr>
<td>25, etc.</td>
<td>220</td>
<td>12</td>
<td>15.49$^6$$^6$,07</td>
<td>49.57</td>
<td>476</td>
<td>50.6</td>
<td></td>
</tr>
</tbody>
</table>

$^1$ Up to $i = 18$: $\Delta t_{i,i+1} = T_{\text{resp.}} = 4 \, \text{s}$. Thereafter the time step increases to $4T_{\text{resp.}} = 16 \, \text{s}$ and $16T_{\text{resp.}} = 64 \, \text{s}$, in order only to reduce the amount of computational work.

$^2$ During the first 12 s breathing takes place via snorkel (14.16 n/l/min), thereafter RMV follows from eq. (2-05).

$^3$ Calculated with eq. (3-07-a).

$^4$ Calculated with eq. (4-13).

$^5$ RMV not applicable as $\delta V_{a,0(+)} = 2.75 \, \text{n}\ell$ is an assumption.
Compilation of equations used in case of scenario II

The computational procedure is shown in fig. 05. Although the calculation proceeds in the same way as in case of scenario I this time however the integration constants $C_{1,i}$ and $C_{2,i}$ must be calculated for each time step again and over again. In order to keep a clear survey, all equations used for the computations, are put together in annex D.

Results dive scenario II-a

The results of scenario II-a are shown in fig. 14. As expected the dive bottle temperature decreases monotonically to the equilibrium sea-water temperature of $26 \degree C$.

Though not important in terms of the numerical implications/progress of the dive bottle temperature, it is clearly seen that the temperature slope changes at $t=0 \, s$, the moment in time when the first inhalation of air from the dive bottle takes place. In the opposite fig. 15 the numerical results are blown up for clarification. The change in the $T_b$-slope is not caused by the decrease in dive air temperature $T_a$ (adiabatic temperature drop), but by the increased heat transfer from sea-water to dive bottle, $h_{w-b}$ which changes from 427 to 705 W/m$^2$K. If this change in $h_{w-b}$ does not take place than the dive bottle temperature progresses as indicated by the dashed curve; $T_b^*$. 

*Note*

As was demonstrated in the earlier analysis of the stylized dives of scenario I, the influence of the process of heat transfer between bottle and contained air hardly has any effect on the temperature progress of the dive bottle. One could erroneously think that the change in the $T_b$-slope is also augmented by the decreasing bottle air temperature. This effect is however nil, extremely small less than $-0.01 \, \degree C$, in case of no breathing at all, $\delta V_{a,i(+)} = 0$.

Noticeable is, that the heat transfer from bottle to the contained air is small, because the air temperature decreases more or less at the same rate as the temperature decrease of the dive bottle. Close until the diver reach bottom depth the air temperature is about equal to the bottle temperature (see fig. 14 at $t = 60 \, s$), implying that the heat transfer between bottle and air diminishes to nil; see for the corresponding change in half value times of air below.

<table>
<thead>
<tr>
<th>Time [s]</th>
<th>-12</th>
<th>0</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
<th>60</th>
<th>72</th>
<th>156</th>
<th>220</th>
<th>540</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_a$ [s]</td>
<td>4642</td>
<td>236</td>
<td>193</td>
<td>186</td>
<td>196</td>
<td>231</td>
<td>1216</td>
<td>381</td>
<td>176</td>
<td>146</td>
<td>114</td>
</tr>
</tbody>
</table>

Consequently, the $T_a$-curve keeps decreasing until it crosses the $T_b$-curve until it approaches a gradually slightly decreasing limit value some degrees below sea-water temperature. This will be elaborated in more detail under scenario II-b.

After the diver reaches bottom depth the inspired air, $\delta V_a$ drops from $6.04$ to $3.1$ nℓ, for two reasons; see fig. 14 upper part. The main reason is that the required amount of work decreases as the swimming velocity of the diver drops down from $23.09$ to $12 \, m/min$, effectively contributing to RMV and thus $\delta V_a$ in the order of the swimming velocity raised to the power of 2.1. Secondly, no air is needed to maintain $V_{tot}$ at $6.5 \, ℓ$, as during descending from 0 to 20 m.

The accumulated decrease in air pressure, $\Delta P_{a,i(+)}$ is, as a matter of fact closely related to the trend in $\delta V_{a,i(+)}$. Recall finally that the later increase of $\delta V_a$ to $12.4$ nℓ and $49.6$ nℓ has nothing to

This value should be infinity. For computation reasons (numerical examination) however at $t = -12 \, s$ a value of $\delta V = 0.001 \, nℓ$ is adopted.
do with the actual respiratory process, but is merely the pay off of reducing the effort of computational work by increasing $T_{\text{resp}}$ from 4 to 16 resp. 64 s (see table 09).

**Results dive scenario II-b**
The results of scenario II-b are shown in fig. 16. As a reminder, this scenario is elaborated only in order to demonstrate the influence of a different initial pressure of dive air; in the present case $P_{a,\text{in}} = 100$ bar instead of 200 bar for scenario II-a. All other input variables remain the same. The influence of the difference in initial air pressure is clearly seen in the fall in $T_a$, during descend due to the larger adiabatic temperature drop; compare lower parts of the figs. 14 and 16. This temperature drop appears to be inversely proportional to the air pressure; see eq. (3-37-a).

For reason already dealt with earlier, the course of the dive bottle temperature is not affected at all by this difference.

The difference in the accumulated dive air pressure **decrease** between II-a and II-b is small, less than 0.8 bar; see upper parts of the figs. 14 and 16. It appears that this difference must be attributed to the isochoric inconsistency of the definition of $\Delta P_a$, with possible erroneous consequences if not properly accounted for; see below for the justification.

**Justification**
The explanation for the difference is brought about by the fact that the dive air temperatures for the cases II-a and II-b are different and therefore must be matched for. Calculated values are:

<table>
<thead>
<tr>
<th>$t$ [s]</th>
<th>II-a (200 bar)</th>
<th>II-b (100 bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_a(0)$ [$^\circ\text{C}$]</td>
<td>25.48</td>
<td>21.11</td>
</tr>
<tr>
<td>$\Delta P_a(0)$ [bar]</td>
<td>14.33</td>
<td>13.47</td>
</tr>
</tbody>
</table>

By definition, $\Delta P_a$ stand for:

$$\Delta P_a = P_{a,\text{initial}} |_{T_a} - P_{a} |_{T_a} \quad (5-02)$$

If $P_{a} |_{T_a}$ is normalized at 32 $^\circ\text{C}$ the corresponding isochoric pressure decrease becomes:

$$\Delta P_{a,32 C} = P_{a,\text{initial}} |_{32 C} - (273.15 + 32) \cdot P_{a} |_{T_a} / (273.15 + T_a(32 ^\circ\text{C})) \quad (5-03)$$

Elimination of $P_{a} |_{T_a}$ from the eqs. (5-02) and (5-03) yields:

$$\Delta P_{a,32 C} = P_{a,\text{initial}} |_{32 C} - (273.15 + 32) \cdot (P_{a,\text{initial}} |_{32 C} - \Delta P_a) / (273.15 + T_a(32 ^\circ\text{C}))$$

or:

$$\Delta P_{a,32 C} = \{(T_a(32 ^\circ\text{C}) - 32) \cdot P_{a,\text{initial}} |_{32 C} + 305.15 \cdot \Delta P_a \} / (273.15 + T_a(32 ^\circ\text{C})) \quad (5-04)$$

Applying eq. (5-04) to case II-a and II-b, yields the following results:

$$\Delta P_{a,32 C, \text{case II-a}} = \{(25.48 - 32) \cdot 200 + 305.15 \cdot 14.33\} / (273.15 + 25.48) = 10.28 \text{ bar} |_{32 C}$$

$$\Delta P_{a,32 C, \text{case II-b}} = \{(21.11 - 32) \cdot 100 + 305.15 \cdot 13.47\} / (273.15 + 21.11) = 10.27 \text{ bar} |_{32 C}$$

Indeed, the normalized pressure decrease in both cases is equal as one should expect, since the air consumption of the diver in both cases is the same. The inconsistent air pressure decreases are once more quoted as 14.33 bar and 13.47 bar for case II-a and II-b. A relative small difference but with relative large implications, if not properly accounted for. If for instance the air pressure decrease is used for estimating the amount of consumed air the answer will be: 172 (14.33 $\cdot V_{\text{db}}$) or 162 nℓ in case of II-a resp. II-b, if the actual dive air temperature is disregarded. However, for both cases it is about 30% less:

$$(273.15 + 20)/(273.15+32)) \cdot 10.3 \cdot V_{\text{db}} = 119 \text{ nℓ at } T_{\text{air}} = 20 ^\circ\text{C}.$$
\[
\text{Lim.} \{\Delta T_{i(+)}\} \equiv T_{b,i} - T_{a,i(+)} = \frac{\tau_{a,i}}{T_{\text{resp},i}} \ln 2 \Delta T_{a,i}
\] (5-05)
in which:
\(\equiv\) stands for: by definition
\text{Lim.} \quad \text{stands for: } T_{a,i+1} - T_{a,i(+)} \rightarrow \Delta T_{a,i+1} \quad (5-06)

In order to avoid possible misunderstandings with regard to the definition of the different quantities, see the opposite fig. 18 for clarification.

Again leaving out the tedious lengthy but elementary mathematical manipulations, eq. (5-05) turns - after eliminating \(\tau_{a,i}\) by means of the set of eqs. (4-38), (4-35-b) and (4-34) - in the following result (note that the numerical "constant"

\[
\text{Lim.} \{\Delta T_{i(+)}\} \approx 23.1 \left(\frac{P_{a,i(+)}[\text{bar}]}{[1+(T_{b,i}[0^\circ C]/273.15)]^{1.1}}\right)^{\frac{1}{4}} \left(\frac{\Delta T_{a,i}[0^\circ C]}{T_{\text{resp},i}[\text{s}]}\right)^{\frac{3}{4}}
\] (5-07)

Turning back to fig. 17 it is seen that, for large values of \(t\) the higher the initial air pressure the smaller the difference/gap between \(T_b\) and \(T_{a,\text{av.}}\). The physical reason is that the larger heat transfer between bottle and contained air causes this effect. Looking to eq. (5-07) one might end up with the opposite observation, as \(\text{lim.} \{\Delta T_{i(+)}\}\) is weakly proportional to \(P_{a,i(+)}\), in fact raised to the power \(\frac{1}{4}\). However this effect is exceeded by the opposite effect that \(\Delta T_{a,i}\) is inversely proportion to the air pressure raised to the power \(\frac{1}{4}\). The net effect will thus be of the order of \(1/\sqrt{\text{initial pressure}}\). Elimination of \(\Delta T_{a,i}\) from eq. (5-07) by means of eq. (3-37-a) results in:

\[
\text{Lim.} \{\Delta T_{i(+)}\} \approx 121 \left(\frac{P_{a,i(+)}}{P_{a,i(-)}^3}\right)^{\frac{1}{4}} \left(\frac{\delta V_{a,i}[\text{nf}]}{T_{\text{resp},i}[\text{s}]}\right)^{\frac{3}{4}} \left(1 + \frac{T_{b,i}[0^\circ C]}{273.15}\right)^{1.225}
\] (5-08)

**Numerical example/clarification**

1. From eq. (5-08) it may be concluded that the order of magnitude of \(\text{lim.} \{\Delta T_{i(+)}\}\) is, approximately inversely proportional to \(\sqrt{P_{a,\text{av.}}}\).
2. Data obtained from fig. 17, at \(t = 540\) s:
   - Note: \(T_b \approx 25.7^\circ C\), because the dive bottle is cooled down by contained cold dive air.
   - \(\text{lim.} \{\Delta T_{540[s]}\}\) \([200\text{bar}] \approx 9.7^\circ C\)
   - \(\text{lim.} \{\Delta T_{540[s]}\}\) \([100\text{bar}] \approx 17.1^\circ C\)
   - \(\text{lim.} \{\Delta T_{540[s]}\}\) \([100\text{bar}] / \text{lim.} \{\Delta T_{540[s]}\}\) \([100\text{bar}] \approx 1.76\)
3. If the above given quantities are estimated by means of eq. (5-08), thereby assuming \(P_{a,i(+)} \approx P_{a,i(-)}\) the following results are obtained, with \(T_{b,i} \approx 26^\circ C\), \(\delta V_{a,i} = 49.57\) nℓ (see table 09), \(T_{\text{resp},i} = 64\) s,
   - \(P_{a,540[s]}\) \([200\text{bar}] = 200 - 50 = 150\text{ bar} \) and \(P_{a,540[s]}\) \([100\text{bar}] \approx 100 - 45 = 55\text{ bar} \):
   - \(\text{lim.} \{\Delta T_{540[s]}\}\) \([200\text{bar}] \approx 9.1^\circ C\)
   - \(\text{lim.} \{\Delta T_{540[s]}\}\) \([100\text{bar}] \approx 15.1^\circ C\)
   - \(\text{lim.} \{\Delta T_{540[s]}\}\) \([100\text{bar}] / \text{lim.} \{\Delta T_{540[s]}\}\) \([100\text{bar}] \approx 1.66\)
   - Obviously the approximations are reasonable accurate.
4. Finally the apparent\(^{25}\) state of equilibrium will be reached after some three times the half value time of dive air, viz. 120 s; thus after some six minutes. This is indeed what can be seen in the \(T_a\) progresses in fig. 17.
5. It is noted that the straight line pieces of \(T_a(t)\) between two successive time indices - see for

\(^{25}\) Apparently, as the \(\text{lim.} \{\Delta T_{i(+)}\}\) values will gradually increase because the decreasing air pressure implies that the heat transfer between bottle to the contained air is decreasing as well.
instance fig 17, lower part) are of an exponential nature in reality; see in this connection the
eqs. (3-41) and (3-42).

5.4 Realistic dive scenarios

Scenario III

There are of course numerous ways to start a dive. In order to get an impression in what way the
pre-phase of the actual descend influences the pressure and temperature progress of the dive
bottle and contained air the following scenario is analyzed:

1. the pre-phase starts with a brief inhalation immediately followed by a deep inhalation, for
   reasons to check the proper functioning of the breathing system: δV_a = 1 + 2½ = 3½ ℓ;
2. after 10 s the diver goes for diving and checks once more his breathing device: δV_a = 1½ ℓ;
3. thereafter the diver swims on snorkel with a velocity of 10 m/min for 60 s to the area where
   the diver intends to descend;
4. next the diver puts his mouthpiece in his mouth and inhales, δV_a = 1½ ℓ, muddles a bit
   around with a swimming velocity of v_d = 7½ m/min during 20 s;
5. than the descend takes place after a final initial inhalation of: δV_a = 2½ ℓ;
6. for the remainder see further down below under parameters.

Fixed data, initial conditions, parameters and assumptions; scenario III

Fixed data

All fixed data is summarized below. To facilitate comparison with the former scenarios, it is
indicated which of these data is also used in the scenarios I and/or II.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>v_des</td>
<td>20 m/min</td>
<td>(as in the scenarios I en II)</td>
</tr>
<tr>
<td>D_bottom</td>
<td>20 m</td>
<td>(as in the scenarios I en II)</td>
</tr>
<tr>
<td>α</td>
<td>30°</td>
<td>(as in the scenarios I en II)</td>
</tr>
<tr>
<td>v_d, des.</td>
<td>23.09 m/min</td>
<td>(as in the scenarios I en II)</td>
</tr>
<tr>
<td>T_w</td>
<td>26 °C</td>
<td>(as in the scenarios I en II)</td>
</tr>
<tr>
<td>T_ref.</td>
<td>273.15 K (0 °C)</td>
<td>(as in the scenarios I en II)</td>
</tr>
<tr>
<td>P_ref.</td>
<td>1 bar</td>
<td>(as in the scenarios I en II)</td>
</tr>
<tr>
<td>V_tot</td>
<td>6½ ℓ</td>
<td>(as in the scenario II)</td>
</tr>
<tr>
<td>T_resp. des.</td>
<td>4 s</td>
<td>(as in the scenarios I and II)</td>
</tr>
</tbody>
</table>

Data on dive bottle and physical-thermal properties of water and air is compiled in annex B

Initial conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_a,-90(-)</td>
<td>200 bar</td>
<td>(as in the scenarios I en II-a)</td>
</tr>
<tr>
<td>T_b,-90</td>
<td>32 °C</td>
<td>(as in the scenarios I en II)</td>
</tr>
<tr>
<td>T_a,-90(-)</td>
<td>32 °C</td>
<td>(as in the scenarios I en II)</td>
</tr>
</tbody>
</table>

Parameters

Two scenarios are considered. Scenario III-a concerns the case where the heat transfer coefficient between sea-water and dive bottle, h_w-b is considered as a parameter in order to account for the uncertainty in this estimated value; see paragraph 4.4 under the heading "Adopted parametric values of h_w-b". During descend the following parametric values for h_w-b, des. are used: 480, 705 and 930 W/m²K. Other values of h_w-b vary in a corresponding fashion, viz. 0.7 times nominal value, nominal value resp. 1.3 times nominal value.

Scenario III-b concerns the case where the heat transfer coefficient between dive bottle and contained air, h_b-a is considered as a parameter in order to account for the uncertainty in this estimated value. As the uncertainty in the estimated value is considered as much larger than in case of h_w-b this time the parameter value of h_b-a are: 0.5 times nominal value, nominal value resp. 1.5 times nominal value. It is not unreasonable to presume that a value of h_b-a of 1.5 times
the nominal value is meaningful to analyse, as the heat transfer from bottle to contained air is certainly increased by the displacements of the air bottle, due to the movements of the diver. The consideration of the case of 0.5 times nominal value is merely included for reasons of parametric consistency.

Compilation of equations used in case of scenario III
See under scenario II and annex D.

Summary of calculation conditions for scenario III
In order to avoid misunderstandings the pre-phase condition of scenario III are depicted here below.

δVₐ [nℓ]:

<table>
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<th>Pre-phase steps [s]</th>
<th>0</th>
<th>10</th>
<th>7½</th>
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</thead>
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<tr>
<td>vₐ [m/min]:</td>
<td>1x10</td>
<td>3x20</td>
<td>2½</td>
</tr>
<tr>
<td>Time [s]:</td>
<td>-90</td>
<td>-80</td>
<td>-20</td>
</tr>
<tr>
<td>Time index, i:</td>
<td>-9</td>
<td>-8</td>
<td>-5</td>
</tr>
</tbody>
</table>

1) Diver swims 'on' snorkel.
2) Estimated with eq. (2.05).
3) As in scenario II.

For more details reference is made to table 10.

Results dive scenario III-a
The results for scenario III-a are shown in fig. 19. In the upper part of the figure only the cumulative dive air pressure decrease is given for case hₜₜₜ= 705 W/m²K, as for the other values the differences are small and only due to the dive air temperature differences; see for more details paragraph 5.3 under the heading 'Results dive scenario II-b, sub Justification'.

In the lower part of fig. 19 the parametric influence of hₜₜₜ on the progress of the dive bottle temperature drop-off, Tₜₜₜ, is clearly visible. On the contrary the influence on the corresponding dive air temperature, Tₐ,i, is weak as the heat transfer from bottle to contained air and vice versa is small. The change in heat transfer from sea-water to dive bottle due to the swimming velocity at the moments in time of -80, -20, 0, and 60 s is seen (red arrows) as a change in the bottle temperature decrease rate.

Compared to scenario II it is noted that the pre-phase, the period before actual diving starts, already resulted in more than half the temperature drop of the dive bottle from 32 to 26 °C. The semi asymptotic values of the dive air temperature estimated by means of eq. (5.08), with δVₐ,i = 24.78 nℓ, Tₜₜₜ,i ≈ 26 °C, Tₑₑₑ = 32 s, ΔTₐ,380s ≈ 1.6 °C and assuming Pₐ,i(+)= Pₐ,i(-)= Pₐ,380(+) | 200 bar = 200 - 36 = 164 bar, is:
lim.ΔTₐ,380(+)| 200 bar ≈ 8.7 °C → lim.Tₐ,av. >380s = 26 - 8.7 + ½ . 1.6 ≈ 18 °C
This estimated value of lim.Tₐ,av. >380s agrees very well with the read-out value from fig. 19.

Results dive scenario III-b
The results for scenario III-b are shown in fig. 20. Only one curve is presented for the course of the dive bottle temperature with time, because the parametric influence of heat transfer from bottle to contained air and vice versa is too small to become apparent; see for that the earlier given considerations on this subject in paragraph 5.3 under the heading "Results dive scenario II-a".
As case III-a with $h_{w-b} = 705 \text{ W/m}^2\text{K}$ is the same as case III-b with $h_{b-a; \text{nominal}}$, fig. 20 shows a large resemblance with fig. 19. For the dive profile, $D$, the inhalation volume, $\delta V_{a,i}$ and the cumulative air pressure decrease, $\Delta P_{a,i}$ see upper part of fig. 19.

For obvious reasons the case, $T_{a,i}$ with $h_{b-a} = 0.5 h_{b-a; \text{nom.}}$ is lagging behind that with $h_{b-a; \text{nom.}}$ and 1.5 $h_{b-a; \text{nom.}}$, implying that at the beginning of the dive (during descend) the air temperature drop-off with 0.5 $h_{b-a; \text{nom.}}$ lies on the right hand side of that with $h_{b-a; \text{nom.}}$, but as time went on the air temperature crosses both curves and lies below them. Compare dotted with the dash-dot $T_{a,i}$-curves in the upper blown up figures in fig. 20.

In the upper right figure it is, as a confirmation seen that the higher the heat transfer coefficient between bottle and contained air the larger the air temperature slope with time during the corresponding time intervals; increasing slope the larger the heat transfer coefficient.

An estimation of the semi asymptotic value of the dive air temperature is obtained with eq. (5-08). However, the constant 121 has to be corrected for those cases where the applied heat transfer coefficient between bottle and contained air deviates from the nominal calculated value, $h_{b-a; \text{nom.}}$. If the deviation is defined as:

$$h_{b-a} \equiv f \cdot h_{b-a; \text{nom.}}$$

than the constant becomes: $121 \cdot f^{-\frac{3}{4}}$ (5-09)

For the present scenario III-b, the following semi asymptotic values of the dive air temperature are than estimated by means of eq. (5-08) and (5-09).

With: $\delta V_{a,i} = 24.78 \text{ nℓ}$, $T_{b,i} \approx 26 \text{ °C}$, $T_{\text{resp}} = 32 \text{ s}$, $\Delta T_{a,380s} \approx 1.6 \text{ °C}$ and assuming that $P_{a,i(+)} \approx P_{a,i(-)} \approx P_{a,380(-)}|200 \text{ bar} = 200 - 36 = 164 \text{ bar}$, one obtains:

$$\lim_{t \to 0.5} \{\Delta T_{380(+)}\} \approx 14.6 \text{ °C} (= 8.7/0.5^{\frac{3}{4}})$$

$$\lim_{t \to 0.5} \{T_{a, \text{av.} >380s}\} = 26 - 14.6 + \frac{1}{2} \cdot 1.6 \approx 12.2 \text{ °C}$$

$$\lim_{t \to 0.5} \{\Delta T_{380(+)}\} \approx 8.7 \text{ °C} \quad \quad \lim_{t \to 0.5} \{T_{a, \text{av.} >380s}\} = 26 - 8.7 + \frac{1}{2} \cdot 1.6 \approx 18.1 \text{ °C}$$

$$\lim_{t \to 1.5} \{\Delta T_{380(+)}\} \approx 6.4 \text{ °C} (= 8.7/1.5^{\frac{3}{4}})$$

$$\lim_{t \to 1.5} \{T_{a, \text{av.} >380s}\} = 26 - 6.4 + \frac{1}{2} \cdot 1.6 \approx 20.4 \text{ °C}$$

This estimated value of $\lim\{T_{a, \text{av.} >380s}\}$ agrees very well with the read-out values from fig. 20.
6 CONCLUSIVE RESULTS AND CONCLUSIONS

Conclusive results

(01) In view of the large amounts of uncertainties in the numerical results, the present report can better be qualified as an assessment than as an reliable answer to the question of what the temperature progress of a dive bottle and the contained air will be during diving, to for example a depth of 20 m.

(02) In answering the above quoted question it appears that a large number of parameters come into play, of which in particular the heat transfer coefficient from (sea-) water to dive bottle (by forced convection) and that from bottle to contained air (caused by natural convection) are at best first estimates.

(03) In order to get some insight in the quantitative effects of the different assumptions and uncertainties the problem at hand is solved by applying a model based approach, viz.:

i. a model for breathing is drafted, or more precisely the inhalation volume per act of breathing in function of the amount of required work (swimming velocity at surface or depth and descend velocity) is established; eqs. (2-05), (3-07-a) and (3-09-b);

ii. a first order (engineering) approach is formulated to quantify the heat transfer process between sea-water and dive bottle on the one hand and between dive bottle and contained air on the other hand; eqs. (4-03) and (4-13) resp. (4-17), (4-18), (4-34), (4-35) and (4-38);

iii. the process of breathing/inspiration is modelled as a series of successive instantaneous inhalations, in order to be able to estimate the air temperature and pressure drop as an adiabatic process; eqs. (3-37-a), (3-28) and (3-38);

iv. a number of dive scenarios are elaborated, the results of which in terms of trends must be conceived and where if possible parametric effects be quantified explicitly.

(04) Three dive scenarios has been chosen to promote insight in the process as such:

I the consideration of a couple of stylized dive scenarios, only in order to conceive/understand trends in the progress of dependent variables with time, including the effects of parameters. The reason behind this scenario lies in the question that numerical calculations might be contaminated with flaws through which mistakes might wait in ambush;

II a couple of simplified dive scenarios, in which the descend to bottom depth takes place immediately after going for diving. As a parameter the initial air pressure in the dive bottle is considered as a parameter;

III a number of realistic dive scenarios are considered, in which the pre-phase of swimming and breathing before descending is introduced and the uncertainty in the establishment of heat transfer from dive bottle to contained air is further assessed.

(05) The thoroughly analytical approach and numerical processing must not be interpreted as yielding accurate results. The heat transfer model is in an analytical sense of an elementary nature indeed; viz. the solution of two simultaneous common - though non-linear - differential equations of the first order with initial boundary conditions. The numerical processing however, demands for much effort because the solution of the set of non-linear differential equations must be performed with small time steps (intervals). These time steps can be chosen as being equal to the breathing rate (period), if in the order of a few seconds, say 4 s. Otherwise the time steps must be chosen in accordance with the intended solution accuracy, which proves to be unnecessary.

(06) The act of inspiration is modelled as taken place during an infinitesimal short period of time, thus allowing the corresponding dive air pressure and temperature drop to treat as an adiabatic process. The breathing period on the other hand is of course finite and need not to be constant.
Relative much time was spent to collect the physical-thermal properties of in particular air (dynamic viscosity and heat conduction coefficient in function of temperature) on the one hand and to resolve the wanted specification of the dive bottle on the other hand.

Conclusions

The model based computational method, used to quantify the temperature progress of the dive bottle and/or the temperature and pressure changes of air contained therein, produces useful results, notwithstanding that the approach is based on a set of elementary models.

The influence of the heat transfer capacity between dive bottle and contained air, on the temperature progress of the dive bottle is very small, less than say 0.2 °C. Reason for that is the small heat capacity of the contained air - at most 14 % of that of the dive bottle - combined with the relative small heat transfer capacity between dive bottle and contained air compared to that of (sea-)water and dive bottle; typical comparable nominal values are: 20 to 30 max. W/m²K resp. 700 W/m²K. This implies that uncertainties in the heat transfer between dive bottle and contained air have hardly any effect on the temperature predictions of the dive bottle.

The half-value time of the temperature progress of the concerned dive bottle at a swimming velocity of the diver of 23 m/min is to the best of the author's knowledge estimated as: $35^{±15}$ s.

With regard to the contained air in the dive bottle, there can only be a question of continuous changing half-value times, as the corresponding heat transfer process is driven by natural convection. During descending to a depth of 20 m the half-value times range from a minimum values in the order of two to three minutes to infinity; see dive scenario II and III.

It is estimated that the semi stationary (asymptotic) dive air temperature is in the order of eight degrees lower than the ambient (sea-)water temperature of 26 °C, under the condition that the diver swims with a velocity of 12 m/min at a depth of 20 m and that the initial air pressure is 200 bar. Numerical and analytically - eqs. (5-07) and (5-08) - it appears that the above given temperature difference increases with decreasing initial air pressure. For instance to 13 to 14 °C in case of an initial air pressure of 100 bar.

If the air pressure in the dive bottle is used as an input quantity to establish the amount of consumed air, than an isochoric pressure correction must be applied to account for the true air temperature in the dive bottle; see for typical examples paragraph 5.3 under the heading "Results dive scenario II-b".

The heat transfer analysis is based on the assumption that the heat conduction capacity through the cylindrical shell of the dive bottle is infinite. Considering the relative small contained air mass, low heat transfer capacity between dive bottle and contained air and the earlier mentioned uncertainties in the heat transfer coefficients all together this simplification need not to be relieved.

An attempt to model in an analytical sense the amount of additional air/oxygen consumption due to the increasing air mass density with depth, has not been successful. Even no conceivable explanation has been found for the thoroughly simple relation that the 'Maximum Breathing Capacity' is about inversely proportional to the root of the air mass density. The reason to try so, was prompted to come across points of departure to model the required air consumption in function of the amount of external work (in the present case for swimming at a certain velocity) with the swimming depth (thus air mass density) as a parameter. In this respect the present breathing model needs to be modified, possibly on bases of the concept of a proper hysteresis model with $\rho_{air}$ as a parameter.

---

26 Technical diving Limited; Luxfer 'site'; aluminium, 12ℓ and 200 bar service pressure.

27 Which is more than five times lower than preliminary guesses mentioned during board meetings of "Stichting Duik-Research".
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Annex A  PARTIAL INCORPORATION OF HEAT CONDUCTION EFFECTS IN THE PROCESS OF HEAT TRANSFER BETWEEN SEAWATER, DIVE BOTTLE AND DIVE AIR DURING SCUBA DIVING

The analysis of the heat transfer process in paragraph 3.3 is based on the assumption that the heat conduction capacity of the cylindrical wall of the dive bottle is that large that the it might be assumed that the wall temperature is homogeneously distributed, viz. constant over the entire wall thickness. A more realistic model is shown in the opposite figure.

So, if heat conduction in the wall of the dive bottle is taken into account, \( T_b \) in the former eqs. (3-10-a) and (3-10-b) must be replaced by \( T_{bw} \) resp. \( T_{ba} \). Resulting in:

\[
\begin{align*}
\frac{dQ}{dt} &= k_b \cdot \frac{T_{bw} - T_{ba}}{t_b} \cdot F_{b, av}. \quad \text{(heat conduction through the wall of the dive bottle)} \\
\text{in which:} & \quad k_b \quad \text{[W/m.K]} \quad \text{heat conduction coefficient of dive bottle} \\
& \quad t_b \quad \text{[m]} \quad \text{cylindrical wall thickness of dive bottle} \\
& \quad F_{b, av.} \quad \text{average heat conductive area of dive bottle} \\
& \quad \text{def. } \frac{1}{2} (F_a + F_b)
\end{align*}
\]

As \( dQ \equiv dQ_2 \), the eqs. (A-02) and (A-03) yields the following solution for \( T_{bw} \) and \( T_{ba} \):

\[
\begin{align*}
T_{bw} &= T_b \pm \frac{t_b}{2 k_b F_{b, av.}} \frac{dQ_2}{dt} \\
T_{ba} &= T_b - \frac{t_b}{2 k_b F_{b, av.}} \frac{dQ_2}{dt}
\end{align*}
\]

Elimination of \( dQ_1 \) and \( dQ_2 \) from the set of eqs. (A-01), (A-05) and (3-11) results into the following set of common simultaneous differential equations:

\[
\begin{align*}
\frac{dT_a}{dt} &= a_{a,k, ef.} \cdot (T_b - T_a) \\
\frac{dT_b}{dt} &= a_b (T_w - T_b) - \lambda_{k, ef.} \cdot a_{a,k, ef.} \cdot (T_b - T_a)
\end{align*}
\]

with, by definition:

\[
\begin{align*}
a_{a,k, ef.} & \equiv \frac{a_a}{1 + k_{\text{effect, air side}}} \\
\lambda_{k, ef.} & \equiv \lambda \cdot (1 + k_{\text{effect, water side}})
\end{align*}
\]

in which: \( a_{a,k, ef.} \quad [1/s] \) as \( a_a \) (eq. (3-13-a), including heat conduction;

\( \lambda_{k, ef.} \quad [-] \) as \( \lambda \) (eq. (3-13-c), including heat conduction;

\[
\begin{align*}
k_{\text{effect, air side}} &= \frac{h_{b-a}}{k_b} \cdot t_b \cdot \frac{F_a}{F_a + F_b} \\
k_{\text{effect, water side}} &= \frac{h_{w-b}}{k_b} \cdot t_b \cdot \frac{F_b}{F_a + F_b}
\end{align*}
\]

In case of the limit transition \( k_b \to \infty \) the set of eqs. (A-06) equals those of (3-16), as expected. For the heat conduction effects, \( k_{\text{effect}} \to 0 \), implying at the same time that the temperature distribution in the dive bottle is homogeneous.
**Bijlage B** SUMMARY OF DATA ON DIVE BOTTLE AND OTHER PHYSICAL- THERMAL PROPERTIES.
Data used to determine heat transfer coefficients.

Dive bottle (assumed type: Luxfer)
Material: aluminium

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comment</th>
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</thead>
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<td>$V_{db}$ [ℓ]</td>
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</tr>
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<td>$d_{ext.}$ [m]</td>
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<td></td>
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<td>$t$ or $t_b$ [mm]</td>
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<tr>
<td>$d_{int.}$ [m]</td>
<td>0.1742</td>
<td>See paragraph 4.2 for justification</td>
</tr>
<tr>
<td>$P_{service}$ [bar]</td>
<td>200 (= 197.4 st. atm.)</td>
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<tr>
<td>$m_b$ [kg]</td>
<td>16.5 (without air)</td>
<td></td>
</tr>
<tr>
<td>$H_{eff.}$ [m]</td>
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<td></td>
</tr>
<tr>
<td>$F_b$ [m$^2$]</td>
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<td></td>
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<tr>
<td>$F_a$ [m$^2$]</td>
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<tr>
<td>$\rho_{b, Al}$ [kg/dm$^3$]</td>
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<td></td>
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<tr>
<td>$c_{b, Al}$ [J/kg·K]</td>
<td>880±10</td>
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<tr>
<td>$k_{Al}$ [W/m·K]</td>
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<tr>
<td>$\beta_{a, 31}$ [K$^{-1}$]</td>
<td>1/ $T_{film}$[K]$^{0.90}$</td>
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<tr>
<td>$\rho_{a}$ [kg/m$^3$]</td>
<td>348</td>
<td></td>
</tr>
</tbody>
</table>
| $\mu_{a}$ [Pa·s] | $1.750 \cdot 10^{-3}$ | Established on basis of data taken from ref. [08]:
| $\mu_{water}$, 0°C [Pa·s] | 1.750 · $10^{-3}$ $T_{water}[/^°C]$ | |
| $\mu_{water}$, 10°C [Pa·s] | 1.294 · $10^{-3}$ | |
| $\mu_{water}$, 20°C [Pa·s] | 0.999 · $10^{-3}$ | |
| $\mu_{water}$, 25°C [Pa·s] | 0.890 · $10^{-3}$ | |
| $\mu_{water}$, 30°C [Pa·s] | 0.798 · $10^{-3}$ | |
| $\mu_{seawater}$, 20°C [Pa·s] | 1.01 · $10^{-3}$ | |

---

1. See eq. (4-27)
2. See eq. (4-26)
3. See eq. (4-25)
4. See eq. (4-24)
5. See eq. (4-23)
6. See eq. (4-22)
7. See eq. (4-21)
8. See eq. (4-20)
9. See ref. [01]
### Table 1

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*Source: Data from Ref. 7.*
**Annex D**  Compilation of equations used in case of Scenario II and III

\[ \text{RMV}_i [\text{lp/min}] = 11.3 + 0.0227 \frac{V_{d,i-1}^{2.1}}{\text{d,i-1}} \quad 0 \leq \text{v}_{d,i-1} [\text{m/min}] \leq 35 \]  

(see eq. (2-05))

or if \( \text{v}_{\text{des}, j} = \text{const} = \text{v}_{\text{des}, i-1} \) and \( \text{T}_{\text{resp}, j} = \text{const} = 4 \text{ s} \):

\[ \delta V_{\text{a,des},i} = \frac{\text{RMV}_i}{60} + \left( \frac{\text{T}_{\text{resp},i-1}}{60} + \frac{\text{RMV}_i}{60} \cdot \sum_{j=0}^{j=i-1} \frac{\text{v}_{\text{des},j} \cdot \text{T}_{\text{resp},j}}{600} \right) \]  

(3-07-a)

\[ \delta V_{\text{a,des},i} = \frac{\text{RMV}_i}{15} + \left( \frac{6.5 + \text{RMV}_i}{15} \cdot \frac{\text{v}_{\text{des},i-1}}{150} \right) \]  

[nl/breathing i]

(3-09-b)

\[ \Delta T_{a,i} \approx 1.22 \frac{(T_{A,i(-)} / 100)^2}{P_{a,i(-)}} \cdot \delta V_{a,i} \]  

(adiabatic temperature drop)  

(see eq. (3-37-a))

\[ T_{A,i(+)} = T_{A,i(-)} - \Delta T_{a,i} \]  

[K]  

(see eq. (3-28))

\[ P_{a,i(+)} = \left( \frac{T_{A,i(+)}}{T_{A,i(-)}} \right)^{3.5} \cdot P_{a,i(-)} \]  

(adiabatic pressure drop)  

(see eq. (3-38))

\[ P_{a,i+1(-)} = \left( \frac{T_{A,i+1(-)}}{T_{A,i(+)}} \right) \cdot P_{a,i(+)} \]  

(isochoric pressure change)  

(see eq. (3-39))

\[ h_{w-b} = 1250 + 400 \cdot (v_d [\text{m/s}])^{0.6} \quad 16 \leq v_d \leq 35 \text{ m/min} \quad [\text{W/m}^2 \cdot \text{K}] \]  

(see eq. (4-13))

\[ \tau_b = 24.1 \cdot 10^3 / h_{w-b} \quad [\text{W/m}^2 \cdot \text{K}] \]  

(see eq. (4-03))

\[ \Delta T_1 = T_{b,i} - T_{A,i(+)} \]  

[°C]  

(see eq. (4-18))

\[ T_{\text{film}, i} = \frac{1}{2} (T_{b,i} + T_{A,i(+)}) \]  

[K]  

(see eq. (4-17))

\[ \text{Gr}_{a, \alpha} = C_{\text{Gr}, \alpha} \cdot 10^{9} \frac{P_{a,i(+)}[^{\text{bar}}]}{(T_{\text{film}, i}[\text{K}]/100)^{4.6}} \]  

[-]  

(see eq. (4-34))

in which:  

\[ C_{\text{Gr}, \text{hor}} = 0.104 \]  

\[ C_{\text{Gr}, 30} = 0.203 \]  

\[ h_{b,a,\text{lam}, i} \approx 9.66 \cdot 10^{-3} \frac{\text{Gr}_{i, \alpha}^{1/4}}{\left( \frac{T_{\text{film}, i}[\text{K}]}{100} \right)^{0.9}} \]  

[W/m² · K]  

(see eq. (4-35-a))

\[ h_{b,a,\text{turb}, i} \approx 2.07 \cdot 10^{-3} \frac{\text{Gr}_{i, \alpha}^{1/3}}{\left( \frac{T_{\text{film}, i}[\text{K}]}{100} \right)^{0.9}} \]  

[W/m² · K]  

(see eq. (4-35-b))

\[ \tau_{a,i} \approx \frac{6.418 \cdot 10^3}{h_{b,a,i} \cdot [\text{W/m}^2 \cdot \text{K}]} \cdot \frac{P_{a,i(+)}[^{\text{bar}}]}{T_{A,i(+)}[\text{K}]} \]  

[s]  

(see eq. (4-38))

\[ \lambda_i = 0.207 \cdot P_{a,i(+)}[^{\text{bar}}] / T_{A,i(+)}[\text{K}] \]  

[-]  

(see eq. (4-41))

\[ C_{1,i} \text{ and } C_{2,i} \]  

[°C]  

(see eqs. (3-22) - (3-27))

\[ T_{b,i+1} \]  

[°C]  

(see eq. (3-21))

\[ T_{a,i+1(-)} \]  

[°C]  

(see eq. (3-20))
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**TABLE 07** Compilation of numerical data; scenario I-a*

<table>
<thead>
<tr>
<th>(\delta V_0)</th>
<th>nℓ</th>
<th>75</th>
<th>75</th>
<th>75</th>
<th>100</th>
<th>100</th>
<th>100</th>
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<td>(10^{12})</td>
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<td>32,3</td>
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<td>0,131</td>
<td>0,131</td>
<td>0,130</td>
<td>0,130</td>
<td>0,130</td>
<td>0,128</td>
<td>0,128</td>
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</tr>
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<td>124,8</td>
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<td>116,8</td>
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<tr>
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</tbody>
</table>

* Initial conditions: \(T_{a,0}\) = \(T_{b,0}\) = 32 °C, \(T_w = 26 \, ^{°}C\), \(P_{a,0}\) = 200 bar.

One inspiration at the moment in time of \(t = 0 \, s\).

Parameter values indicated in blue row are: breathing volume \(\delta V_0 \, [nℓ]\) and thermal heat transfer coefficient \(h_{w-b} \, [W/m^2K]\).

**Note**  
Read all comma’s as points.

[Document: SDR_Temp lucht en fles_Gestileerde duik I-a vsE[stick-02]]
**TABLE 08-1/2**  Compilation of numerical data; scenario I-b

*Time interval 0< t[s] < 30(-)*

<table>
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<tr>
<th>(\partial V_0)</th>
<th>n(t)</th>
<th>(\Delta T_{a,0})</th>
<th>(T_{a,0(+)})</th>
<th>(P_{a,0(+)})</th>
<th>(\Delta P_{a,0})</th>
<th>(T_{0})</th>
<th>(\Delta T_{0})</th>
<th>(h_{w-b})</th>
<th>(G_{r,32})</th>
<th>(h_{b,a,turb})</th>
<th>(\tau_{a,1})</th>
<th>(\lambda_0)</th>
<th>(T_{b})</th>
<th>(T_{b,30})</th>
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<td>26 + 6 x 2^(-t/34,43)</td>
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</tr>
</tbody>
</table>

* Initial conditions: \(T_{a,0(+)} = T_{b,0} = 32 \degree C\) en \(P_{a,0(+)} = 200\) bar.
  One inhalation at the moment in time of \(t = 30\) s.
  \(h_{w-b} = 700\) W/m²K in all cases of scenario I-b.

**Note**  Read all comma’s as points.
**TABLE 08-2/2**  Compilation of numerical data; scenario I-b*

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<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
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<td>0.130</td>
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<td>0.00014</td>
<td>0.00021</td>
<td>0.00027</td>
<td>0.00016</td>
<td>0.00024</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>s</td>
<td>256.5</td>
<td>181.5</td>
<td>141.3</td>
<td>207.0</td>
<td>147.0</td>
<td>115.0</td>
<td>182.0</td>
<td>129.6</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>s</td>
<td>33.7</td>
<td>33.4</td>
<td>33.0</td>
<td>33.5</td>
<td>33.1</td>
<td>32.5</td>
<td>33.4</td>
<td>32.8</td>
</tr>
<tr>
<td>$F_1$</td>
<td>-</td>
<td>0.020</td>
<td>0.031</td>
<td>0.042</td>
<td>0.026</td>
<td>0.040</td>
<td>0.055</td>
<td>0.030</td>
<td>0.046</td>
</tr>
<tr>
<td>$C_{l,31}$</td>
<td>0°C</td>
<td>2.241</td>
<td>2.490</td>
<td>2.759</td>
<td>0.969</td>
<td>1.307</td>
<td>1.682</td>
<td>-0.333</td>
<td>0.085</td>
</tr>
<tr>
<td>$C_{l,31}$</td>
<td>0°C</td>
<td>-0.501</td>
<td>-0.750</td>
<td>-1.019</td>
<td>-0.649</td>
<td>-0.987</td>
<td>-1.362</td>
<td>-0.767</td>
<td>-1.185</td>
</tr>
<tr>
<td>$C_{l,31} \times F_1$</td>
<td>0°C</td>
<td>0.046</td>
<td>0.077</td>
<td>0.117</td>
<td>0.025</td>
<td>0.052</td>
<td>0.093</td>
<td>-0.010</td>
<td>0.004</td>
</tr>
<tr>
<td>$C_{l,31} \times F_2$</td>
<td>0°C</td>
<td>3.234</td>
<td>3.203</td>
<td>3.163</td>
<td>3.255</td>
<td>3.228</td>
<td>3.187</td>
<td>3.290</td>
<td>3.276</td>
</tr>
</tbody>
</table>

* Initial conditions: $T_{a,0} = T_{b,0} = 32$ °C en $P_{a,0} = 200$ bar.
  One inhalation at the moment in time of $t = 30$ s.
  $h_{w,b} = 700$ W/m²K in all cases of scenario I-b.
  Parameter values indicated in blue row are: breathing volume $\delta V_{30}$ [nℓ] and thermal heat transfer coefficient $h_{b,a}$ [W/m²K].

**Note**  Read all comma's as points.

[Document: SDR_Temp lucht en flies_Gestileerde duik I-b vsE [stick-02]]

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### Table 10: Summary of computational basis for dive scenario III.

Parameter value: $h_{w-b, dse} = 705 \text{ W/m}^2\text{K}$ and $h_{b-a}$ nominal calculated values.

<table>
<thead>
<tr>
<th>i [-]</th>
<th>$t_i$ [s]</th>
<th>$\Delta t_{i+1}$ [s]</th>
<th>$v_{d, i/i+1}$ [m/min]</th>
<th>$\text{RMV}_{i/i+1}$ [nℓ/min]</th>
<th>$\delta V_{i,i+1}$ [nℓ]</th>
<th>$h_{w-b, i}$ [W/m$^2$K]</th>
<th>$\tau_{b, i}$ [s]</th>
<th>Type of flow</th>
<th>$\tau_{a, i}$ [s]</th>
<th>$h_{b-a, i}$ [W/m$^2$K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9</td>
<td>-90</td>
<td>10 1)</td>
<td>n.a.</td>
<td>3½ 2)</td>
<td>= 0</td>
<td>∞</td>
<td>439</td>
<td>9.6</td>
<td>9.6</td>
<td>9.6</td>
</tr>
<tr>
<td>-8</td>
<td>-80</td>
<td>20 1)</td>
<td>10 2)</td>
<td>n.a.</td>
<td>1½ 2)</td>
<td>427</td>
<td>56.5</td>
<td>10.7</td>
<td>10.7</td>
<td>10.7</td>
</tr>
<tr>
<td>-7,... till -5</td>
<td>60, etc.</td>
<td>20</td>
<td>10 0 3)</td>
<td>0 3)</td>
<td>427</td>
<td>56.5</td>
<td>255, ...</td>
<td>14.1</td>
<td>16.5</td>
<td>18.2</td>
</tr>
<tr>
<td>-5</td>
<td>-20</td>
<td>7½ 2)</td>
<td>12.9</td>
<td>1½ 2)</td>
<td>359</td>
<td>67.1</td>
<td>188</td>
<td>20.1</td>
<td>22.3</td>
<td>23.7</td>
</tr>
<tr>
<td>-4,... till 0</td>
<td>-16, etc.</td>
<td>4</td>
<td>7½ 4)</td>
<td>12.9</td>
<td>0.86</td>
<td>359.</td>
<td>188, ...</td>
<td>20.1</td>
<td>22.3</td>
<td>23.7</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>4</td>
<td>23.09</td>
<td>27.87</td>
<td>705</td>
<td>34.2</td>
<td>Turbulent, Dive bottle horizontal.</td>
<td>151</td>
<td>25.0</td>
<td>27.7</td>
</tr>
<tr>
<td>1,... till 14</td>
<td>4, etc.</td>
<td>4</td>
<td>23.09</td>
<td>27.87</td>
<td>705</td>
<td>34.2</td>
<td>Laminar 8)</td>
<td>1115</td>
<td>14.8</td>
<td>3.6</td>
</tr>
<tr>
<td>14</td>
<td>56</td>
<td>4</td>
<td>23.09</td>
<td>27.87</td>
<td>705</td>
<td>34.2</td>
<td>Turbulent, Dive bottle horizontal.</td>
<td>370</td>
<td>14.1</td>
<td>10.9</td>
</tr>
<tr>
<td>15</td>
<td>60</td>
<td>4</td>
<td>12 2)</td>
<td>27.87</td>
<td>6.44</td>
<td>476</td>
<td>50.6</td>
<td>328, ...</td>
<td>14.9</td>
<td>12.3</td>
</tr>
<tr>
<td>16,... till 19</td>
<td>64, etc.</td>
<td>4</td>
<td>12 3)</td>
<td>15.49</td>
<td>3.10</td>
<td>476</td>
<td>50.6</td>
<td>264</td>
<td>16.8</td>
<td>15.2</td>
</tr>
<tr>
<td>19</td>
<td>76</td>
<td>16 3)</td>
<td>12</td>
<td>15.49</td>
<td>3.10</td>
<td>476</td>
<td>50.6</td>
<td>223, ...</td>
<td>18.8</td>
<td>17.9</td>
</tr>
<tr>
<td>20,... till 24</td>
<td>92, etc.</td>
<td>16</td>
<td>12</td>
<td>15.49</td>
<td>12.39</td>
<td>476</td>
<td>50.6</td>
<td>165</td>
<td>23.7</td>
<td>23.7</td>
</tr>
<tr>
<td>24</td>
<td>156</td>
<td>32 3)</td>
<td>12</td>
<td>15.49</td>
<td>12.39</td>
<td>476</td>
<td>50.6</td>
<td>152, ...</td>
<td>25.2</td>
<td>25.4</td>
</tr>
<tr>
<td>25,... till 31</td>
<td>188, etc.</td>
<td>32</td>
<td>12</td>
<td>15.49</td>
<td>24.78</td>
<td>476</td>
<td>50.6</td>
<td>123</td>
<td>28.6</td>
<td>28.8</td>
</tr>
<tr>
<td>31</td>
<td>380</td>
<td>End of scenario III</td>
<td>24.78</td>
<td>End of scenario III.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1) Chosen computational value.

2) Assumed value for this scenario.

3) Diver swims on snorkel.

4) Estimated value with eq. (4-13).

5) Calculated with eq. (2-05).

6) Calculated with eq. (3-07-a) or (3-09-b).

7) Calculated with eq. (4-03).

8) And $\alpha = 30^0$.

9) Calculated with the set of equations summarized in annex D; eq.(4-38).

10) $h_{b-a}$ calculated with the set of eqs. (4-34) and (4-35) for scenario III-a with $h_{w-b}$ as parameter: 0.7 x nom., nom. and 1.3 x nom.
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<thead>
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<th>PAGE</th>
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<td>Figure 02 RMV in function of swimming velocity.</td>
<td>12</td>
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<tr>
<td>Figure 03 RMV in function of OMV.</td>
<td>13</td>
</tr>
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<td>13</td>
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<td>Figure 07 Temperature progress of dive air ($T_a$) and bottle ($T_b$) and air pressure decrease ($\Delta P_a$) in case of the stylized dive with one inspiration of <strong>75 nℓ</strong> only at the commencement of the dive (scenario I-a).</td>
<td>67</td>
</tr>
<tr>
<td>Figure 08 Temperature progress of dive air ($T_a$) and bottle ($T_b$) and air pressure decrease ($\Delta P_a$) in case of the stylized dive with one inspiration of <strong>100 nℓ</strong> only at the commencement of the dive (scenario I-a).</td>
<td>68</td>
</tr>
<tr>
<td>Figure 09 Temperature progress of dive air ($T_a$) and bottle ($T_b$) and air pressure decrease ($\Delta P_a$) in case of the stylized dive with one inspiration of <strong>125 nℓ</strong> only at the commencement of the dive (scenario I-a).</td>
<td>69</td>
</tr>
<tr>
<td>Figure 10 $T_a$ and $T_b$ in function of time in case of scenario I-a with $\delta V_0 = 75$ nℓ and $h_{w-b} = 700$ W/m$^2$K.</td>
<td>36</td>
</tr>
<tr>
<td>Figure 11 Temperature progress of dive air ($T_a$) and bottle ($T_b$) in case of the stylized dive with one inspiration only, of 75, 100 or 125 nℓ, 30 s after the descending (scenario I-b).</td>
<td>70</td>
</tr>
<tr>
<td>Figure 12 $T_b$, $T_a$ and $\Delta T_b$ in function of time in case of scenario I-b with $\delta V_{30} = 100$ nℓ, $h_{w-b} = 700$ W/m$^2$K and $h_{b-a}$ as parameter (with extreme values).</td>
<td>38</td>
</tr>
<tr>
<td>Figure 13 $T_b$ and $T_a$ in function of time in case of scenario I-b with $\delta V_{30} = 100$ nℓ, $h_{w-b} = 700$ W/m$^2$K and $h_{b-a}$ as parameter.</td>
<td>39</td>
</tr>
<tr>
<td>Figure 14 Dive bottle pressure decrease, breathing/inhalation volume, temperature of dive bottle and of the air therein in function of time, for a simplified dive with nominal calculated heat transfer coefficients and with $P_{b,initial} = 200$ bar (dive scenario II-a).</td>
<td>71</td>
</tr>
<tr>
<td>Figure 15 $T_b$ and $T_a$ in function of time in case of scenario II-a (blown-up part of fig. 14).</td>
<td>41</td>
</tr>
<tr>
<td>Figure 16 As fig. 14, but with $P_{b,initial} = 100$ bar (dive scenario II-b)</td>
<td>72</td>
</tr>
<tr>
<td>Figure 17 Dive bottle pressure decrease, breathing/inhalation volume, temperature of dive bottle and of the contained air in function of time, for a simplified dive with nominal calculated heat transfer coefficients and with $P_{b,initial}$ as parameter; 200 and 100 bar (dive scenario II-a and II-b).</td>
<td>73</td>
</tr>
<tr>
<td>Figure 18 Clarification of the quantities $T_{a,i}$, $\Delta T_{a,i+1}$ and $\Delta T_{i+1}$.</td>
<td>43</td>
</tr>
<tr>
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<td>74</td>
</tr>
<tr>
<td>Figure 20 Dive bottle and dive air temperature progress in function of time with $h_{b-a}$ as a parameter for the realistic dive scenario III-b</td>
<td>75</td>
</tr>
</tbody>
</table>
Fig. 07 Temperature progress of dive air \((T_a)\) and bottle \((T_b)\) and air pressure decrease \((\Delta P_a)\) in case of the stylized dive with **one inspiration of 75 nl only** at the commencement of the dive (scenario I-a).

**Note** In this figure the heat transfer coefficient between water and dive bottle is incorporated as parameter; \(h_{w-b} = 480, 700\) resp. 930 W/m\(^2\)K. Initial conditions at the moment in time of \(t = 0\): \(T_b = T_a = 32^\circ\text{C}, T_w = 26^\circ\text{C}\) and \(P_{a,0(c)} = 200\) bar.
Fig. 08 Temperature progress of dive air ($T_a$) and bottle ($T_b$) and air pressure decrease ($\Delta P_a$) in case of the stylized dive with one inspiration of 100 nl only at the commencement of the dive (scenario I-a).

*Note*  In this figure the heat transfer coefficient between water and dive bottle is incorporated as a parameter; $h_{w,b} = 480, 700 \text{ resp. } 930 \text{ W/m}^2\text{K}$.

Initial conditions at the moment in time of $t = 0$: $T_b = T_a = 32 ^\circ \text{C}$, $T_w = 26 ^\circ \text{C}$ and $P_{a,0} = 200 \text{ bar}$. 

[Document: SDR_Temp lucht en fles_Gestileerde duik I-a vsE [stick-02]]

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Fig. 09 Temperature progress of dive air ($T_a$) and bottle ($T_b$) and air pressure decrease ($\Delta P_a$) in case of the stylized dive with **one inspiration of 125 nℓ only** at the commencement of the dive (scenario I-a).

**Note** In this figure the heat transfer coefficient between water and dive bottle is incorporated as a parameter; $h_{w,b} = 480, 700$ resp. $930$ W/m$^2$K. Initial conditions at the moment in time of $t = 0$: $T_b = T_a = 32$ °C, $T_w = 26$ °C and $P_{a,0(\text{in})} = 200$ bar.
Fig. 11  Temperature progress of dive air (Tₐ) and bottle (Tₖ) in case of the stylized dive with **one inspiration** only of 75, 100 or 125 ℓ, 30 s after the beginning of the dive (scenario I-b).

**Note** In each figure the *heat transfer coefficient of bottle to air* has been presented as a parameter, on the understanding that the nominal calculated values (23.1, 28.5 resp. 32.1 W/m²K; red in bold), in view of their uncertainties have been multiplied with 0.7 resp. 1.3. Initial conditions at the moment in time of t = 0: Pₐ = 200 bar, Tₐ = Tₖ = 32 °C and Tₖ = constant = 26 °C.
Fig. 14  Dive bottle pressure decrease, breathing/inhalation volume, temperature of dive bottle and of air contained therein in function of time, for a simplified dive with nominal calculated heat transfer coefficients and with $P_{b, initial} = 200$ bar (dive scenario II-a).

[Brondocument SDR_Temp fles en lucht_scenario II-a en II-b vsE [stick-02]].
Fig. 16  As fig. 14, but with \( P_{b, \text{initial}} = 100 \) bar (dive scenario II-b).
Fig. 17  Dive bottle pressure decrease, breathing/inhalation volume, temperature of dive bottle and of the contained air therein in function of time, for a simplified dive with nominal calculated heat transfer coefficients with $P_b, \text{initial}$ as parameter; 200 and 100 bar. (dive scenario II-a and -b).
Fig. 19 Progress in dive bottle pressure decrease, breathing/inhalation volume, temperature of dive bottle and of contained air in function of time for the realistic dive scenario III-a with $h_{w-b}$ as parameter.
Fig. 20 Dive bottle and dive air temperature progress in function of time with $h_{b-a}$ as a parameter for the realistic dive scenario III-b.
### LIST OF SYMBOLS

**Latin**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit/Dimension</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[1/s]</td>
<td>combined thermal decay 'constant' of dive bottle and contained air; eq. (3-25).</td>
</tr>
<tr>
<td>a</td>
<td>[1/s]</td>
<td>thermal decay 'constant'; eq. (3-17) and (3-18).</td>
</tr>
<tr>
<td>B</td>
<td>[1/s²]</td>
<td>product of thermal decay 'constant' of dive bottle and contained air; eq. (3-25).</td>
</tr>
<tr>
<td>B</td>
<td>[kg]</td>
<td>buoyancy force.</td>
</tr>
<tr>
<td>C_i</td>
<td>[°C]</td>
<td>integration coefficient with regard to the time interval t_i-1 → t; eq. (3.22).</td>
</tr>
<tr>
<td>C_j</td>
<td>[°C]</td>
<td>integration coefficient with regard to the time interval t_i-1 → t; eq. (3.23).</td>
</tr>
<tr>
<td>CMV</td>
<td>[nℓ/min]</td>
<td>&quot;Carbon dioxide minute Value&quot;.</td>
</tr>
<tr>
<td>c</td>
<td>[J/kg °C]</td>
<td>specific heat capacity.</td>
</tr>
<tr>
<td>D</td>
<td>[m]</td>
<td>bottom depth or characteristic diameter in dimensionless groups.</td>
</tr>
<tr>
<td>F</td>
<td>[m²]</td>
<td>heat transfer area of dive bottle.</td>
</tr>
<tr>
<td>F_1</td>
<td>[-]</td>
<td>integration coefficient; eq. (3-27).</td>
</tr>
<tr>
<td>F_2</td>
<td>[-]</td>
<td>integration coefficient; eq. (3-27).</td>
</tr>
<tr>
<td>F_3</td>
<td>[-]</td>
<td>integration coefficient; eq. (3-27).</td>
</tr>
<tr>
<td>Gr</td>
<td>[-]</td>
<td>number of Grashof; eq. (4-07).</td>
</tr>
<tr>
<td>g</td>
<td>[m/s²]</td>
<td>gravitational acceleration.</td>
</tr>
<tr>
<td>H</td>
<td>[dm of m]</td>
<td>length/height of dive bottle.</td>
</tr>
<tr>
<td>h</td>
<td>[W/m²K]</td>
<td>heat transfer coefficient.</td>
</tr>
<tr>
<td>k</td>
<td>[W/mK]</td>
<td>heat conduction coefficient.</td>
</tr>
<tr>
<td>L</td>
<td>[m]</td>
<td>characteristic length.</td>
</tr>
<tr>
<td>m</td>
<td>[kg]</td>
<td>mass.</td>
</tr>
<tr>
<td>Nu</td>
<td>[-]</td>
<td>number of Nusselt; eq. (4-04).</td>
</tr>
<tr>
<td>OMV</td>
<td>[nℓ/min]</td>
<td>&quot;Oxygen Minute Value&quot;.</td>
</tr>
<tr>
<td>P</td>
<td>[bar]</td>
<td>pressure.</td>
</tr>
<tr>
<td>P</td>
<td>[Nm/s]</td>
<td>power.</td>
</tr>
<tr>
<td>Pr</td>
<td>[-]</td>
<td>number of Prandtl; eq. (4-05).</td>
</tr>
<tr>
<td>ΔP_a</td>
<td>[bar]</td>
<td>adiabatic air pressure drop in dive bottle brought about by an instantaneous inspiration; eq. (3.38).</td>
</tr>
<tr>
<td>Q</td>
<td>[J]</td>
<td>amount of transported heat; eqs. (3-10) and (3-11).</td>
</tr>
<tr>
<td>Re</td>
<td>[-]</td>
<td>number of Reynolds; eq. (4-06).</td>
</tr>
<tr>
<td>RMV</td>
<td>[nℓ/min]</td>
<td>&quot;Respiratory Minute Value&quot;.</td>
</tr>
<tr>
<td>RQ</td>
<td>[-]</td>
<td>Respiratory Quotient.</td>
</tr>
<tr>
<td>T</td>
<td>[K of °C]</td>
<td>temperature.</td>
</tr>
<tr>
<td>T_A</td>
<td>[K]</td>
<td>air temperature in K.</td>
</tr>
<tr>
<td>T_film</td>
<td>[K]</td>
<td>film temperature, being the average value of the dive bottle temperature and that of the contained dive air; eq. (4-17).</td>
</tr>
<tr>
<td>Tresp,i</td>
<td>[min]</td>
<td>breathing period, viz. time interval between two successive moments of instantaneous inspiration at t_i-1 and t_i; see fig. 01. Also used as time step, for the computations on the progress of the transient heat transfer process.</td>
</tr>
<tr>
<td>ΔT</td>
<td>[K or °C]</td>
<td>difference in dive bottle temperature and temperature of contained dive air; see fig. 18 and eq. (4-18).</td>
</tr>
<tr>
<td>ΔT_a</td>
<td>[K or °C]</td>
<td>adiabatic air temperature drop in dive bottle brought about by an instantaneous inspiration; eqs. (3-28) en (3-37).</td>
</tr>
<tr>
<td>t</td>
<td>[s or min]</td>
<td>time.</td>
</tr>
<tr>
<td>t</td>
<td>[mm]</td>
<td>wall thickness of dive bottle.</td>
</tr>
<tr>
<td>V</td>
<td>[pℓ or nℓ]</td>
<td>volume (at ambient pressure pℓ resp. atmospheric pressure nℓ).</td>
</tr>
<tr>
<td>V_res.</td>
<td>[ℓ]</td>
<td>residual lung volume.</td>
</tr>
<tr>
<td>V_tot.</td>
<td>[ℓ]</td>
<td>total long capacity.</td>
</tr>
<tr>
<td>V_urv</td>
<td>[ℓ]</td>
<td>unused reserve lung volume; eq. (2-02).</td>
</tr>
</tbody>
</table>
$V_{\text{work}}$ [ℓ] required tidal volume, in order to be able to perform a certain amount of work.

$\delta V$ [ℓ] instantaneous required inspiration volume; eq. (3-07).

$v$ [m/s] velocity.

$v_d$ [m/min] swimming velocity of scuba diver.

$v_{\text{des.}}$ [m/min] descend velocity of scuba diver.

Greece

$\beta$ [m$^3$/m$^3$°C] volumetric thermal expansion coefficient.

$\gamma$ [-] ratio of specific heat capacity at constant pressure and that at constant volume.

$\lambda$ [-] ratio of heat capacity of dive air and that of dive bottle; eqs. (3-19), (4-41) and (4-42).

$\mu$ [Pa.s] dynamic viscosity.

$\nu$ [m$^3$/s] kinematic viscosity, $\nu = \mu/\rho$.

$\nu_a$ [1/s] breathing frequency.

$\rho$ [kg/m$^3$] specific mass.

$\tau$ [s] half-value time.

$\tau_1$ [s] computational half-value time; eq. (3-26).

$\tau_2$ [s] computational half-value time; eq. (3-26).

$\psi$ [m$^3$/s] volume rate.

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A or a dive air.

alv. alveoli.

amb. ambient.

b or db dive bottle.

des. descend.

d.s. dead space.

eff. effective ...

ext. external.

i breathing or time index.

int. internal.

i(−) value at the moment of time $t_i = \lim_{t \to t_i} \{t \to t_i\}$.

i(+) value at the moment of time $t_i = \lim_{t \to t_i} \{t \to t_i\}$.

p at constant pressure.

part. partial ...

ref. reference value of ...

v at constant volume.

w sea-water